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An empirical evaluation of structural credit risk models

by Nikola A Tarashev*

Monetary and Economic Department

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An Empirical Evaluation of Structural Credit Risk Models[①]

Nikola A Tarashev[②]

Bank for International Settlements

Abstract

This paper evaluates empirically the performance of six structural credit risk models by comparing the probabilities of default (PDs) they deliver to ex post default rates. In contrast to previous studies pursuing similar objectives, the paper employs firm-level data and finds that theory-based PDs tend to match closely the actual level of credit risk and to account for its time path. At the same time, non-modelled macro variables from the financial and real sides of the economy help to substantially improve the forecasts of default rates. The finding suggests that theory-based PDs fail to fully reflect the dependence of credit risk on the business and credit cycles. Most of the upbeat conclusions regarding the performance of the PDs are due to models with endogenous default. For their part, frameworks that assume exogenous default tend to under-predict credit risk. Three borrower characteristics influence materially the predictions of the models: the leverage ratio; the default recovery rate; and the risk-free rate of return.

Key words: probability of default, credit risk models, Basel II, macroeconomic factors of credit risk

JEL Classification Numbers: C52, G1, G3

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② nikola.tarashev@bis.org

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Introduction

Predictors of credit (or default) risk, ie the risk that a borrower does not fulfil its debt contract, are of natural interest to practitioners in the financial industry as well as to regulators. The accuracy of these predictors is essential for sound risk management and for supervisory evaluation of the vulnerability of lender institutions. In an appreciation of this, the new capital adequacy framework (Basel II) encourages the active involvement of banks in measuring the likelihood of defaults. The growing need for reliable measures of credit risk prompts the question whether they can be obtained from academic theoretical models.

The finance literature has produced a variety of models that attempt to measure default risk. In this paper, I consider the family of *structural* models, which focus on the stochastic process of a corporate obligor's assets and postulate that a default occurs when these assets cross a threshold value. The models can be divided into an "endogenous default" and an "exogenous default" group. The frameworks in the former group let borrowers choose strategically the timing of default.[1] In contrast, the models in the latter group impose an *ad hoc* default trigger but develop richer stochastic structures that capture empirical regularities of credit markets.

I focus on one component of default risk, the probability of default (PD),[2] and attempt to answer the question: *How do structural credit risk models fare when put to the test of the data?* In general terms, my conclusions rely on comparisons between firm-specific model-based PDs of corporate borrowers and the corresponding *ex post* default rates.

The exercise is carried out in the context of two "endogenous default" models, those developed in Leland and Toft (1996) (henceforth, LT) and Anderson, Sundaresan and Tychon (1996) (AST), and three "exogenous default" frameworks, developed in Longstaff and Schwartz (1995) (LS), Collin-Dufresne and Goldstein (2001) (CDG) and Huang and Huang (2003) (HH). As benchmarks, I also consider PDs delivered by the model of the commercial service Moody's KMV (henceforth, MKMV). The latter framework is not publicly available but is known to share key features of the "academic" models, to use proprietary information on credit outlooks and to be estimated on the basis of historical default rates.

One of the main conclusions of the paper is that, in general, theory-based PDs track closely ex post default rates and do so for different forecast horizons. The best performers are the "endogenous default" models, which feature virtually unbiased forecasts. In contrast, the "exogenous default" frameworks tend to underpredict default rates. Considering the entire sample, the bias is small under the "exogenous default" LS and HH models but is quite pronounced under the CDG model. The

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[1] The terms "debtor", "obligor" and "borrower" are used interchangeably throughout the paper.

[2] Other components of default risk, such as loss given default and exposure at default, are not analysed in this paper.

MKMV PDs are found to be the highest across models and to generally overpredict default risk. Nevertheless, owing to the short available time series of these PDs, the result remains inconclusive.

The finding that "academic" models closely match the overall level of default rates stands in sharp contrast to a conclusion of Leland (2002). That paper calibrates the LT and LS models to the *representative* borrower and concludes that they substantially underestimate ex post default rates at short horizons, such as one year: the horizons typically used in practical applications. The stark differences between the findings of this paper and the conclusions of Leland (2002) are due to the strongly non-linear relationships between inputs to the models (ie parameter values) and their implications for PDs. The non-linearities are such that a high theory-based PD is much more sensitive to parameter changes than a low PD. This gives rise to the so-called "Jensen inequality" effect, whereby the average theoretical PD across borrowers (which is used here and is an unbiased estimator of default rates if its underlying model is correct) is larger than the theoretical PD of the average (or representative) borrower (used in Leland (2002)).

I also examine the *economic* significance of the errors in theoretical forecasts of default rates. To do so, I focus on the foundation IRB approach of Basel II, which provides a formula for mapping a PD into minimum capital requirements.[3] Using the IRB formula, I compare model-implied capital with the "optimal" capital, which is based on the "true" PDs revealed ex post. Adopting perfect knowledge of credit risk as a benchmark, the exercise helps appreciate the economic costs of relying on structural models for regulatory purposes.

The results of this stylised exercise reveal a mixed message. At one extreme, the regulatory capital implied by the "endogenous default" models tracks extremely closely the "optimal" capital level. In particular, when filtered through the IRB approach, the forecast errors of these models represent a small fraction of the average level and volatility of the capital requirements under perfect knowledge of credit risk. At the other extreme, the "exogenous default" models provide economically significant forecast errors and perform more poorly than even the simplest, standardised, approach of Basel II, which does not consider explicitly PD estimates but relies exclusively on external credit ratings.

Finally, I analyse the capacity of the structural models to explain the evolution of credit risk over time. In particular, I examine the significance of theoretical PDs as regressors of ex post default rates. As control variables, I consider macroeconomic indicators reflecting the real and financial sides of the economy. These variables are motivated by Estrella and Hardouvelis (1991) and Smets and Tsatsaronis (1997), who identify predictors of economic activity, and by Borio and Lowe (2002), who identify predictors of banking system distress.

The results of the regression analysis indicate that the structural models reveal useful information regarding the time pattern of default rates. The message is stronger when the theoretical implications are based on a time invariant estimate of the risk-free rate because the "academic" models tend to

[3] See Basel Committee on Banking Supervision (2004). "IRB" stands for "internal ratings based".

associate rises in that rate with lower credit risk, which is not supported by the data. In addition, using both "endogenous" and "exogenous default" models, as opposed to a single model from either group, improves substantially the forecasts of default rates. Nevertheless, the macroeconomic controls *do* add to and, in certain asset classes even substitute for, the informational content of the models. The finding reveals that theoretical PDs fail to fully reflect the dependence of credit risk on the business and credit cycles.[4] A parallel analysis of the MKMV PDs conveys a similar message.

Disparities in the performance of different academic models can be explained by the way they handle two key debt characteristics: the default-trigger value of assets and the dead-weight cost incurred at the time of default. Under the "endogenous default" models, these characteristics are to be in line with a market-wide estimate of the default recovery rate as well as with a number of other borrower-specific features, the most important of which turns out to be firm leverage.[5] This leads to a strong dispersion of the two debt characteristics across borrowers and, by the "Jensen inequality" effect, raises the theoretical predictions to levels that closely match the observed default rates. In addition, the "endogenous default" models imply that the default boundary increases in the default cost, which is in accord with the empirical regularity that default rates and losses given default tend to move in the same direction. By contrast, the "exogenous default" models suggest ad hoc values for both the default boundary and the default cost. In such a context, the two debt characteristics are also calibrated to be consistent with the default recovery rate but depend little on borrower-specific features. This underplays differences in the credit outlooks across firms, which leads to an underestimation of default rates.

Besides the aforementioned Leland (2002), several other articles have also evaluated the empirical performance of structural credit-risk models but from a different perspective. Huang and Huang (2003), for example, calibrate the models to observed default rates and then study the implied credit risk premiums. In a more recent study, Eom, Helwege and Huang (2004) use data on individual bonds in order to compare their credit spreads to the predictions of structural credit risk models. While these articles examine theoretical pricing implications, which are based on preference-weighted probabilities that reflect bond holders' risk appetite, the analysis herein focuses on actual (or statistical) PDs.

The rest of the paper is organised as follows. In Section 1, I present the six models and highlight assumptions that are expected to have strong implications for the implied PDs. Then, in Section 2, I describe the data and, in Section 3, outline how they are employed for the calibration of the models. In Section 4, I present the theoretical PDs and explain differences among them on the basis of the underlying modelling assumptions. In Section 5, I consider the implications of the models for banks' minimum capital requirements. Finally, in Section 6, I test the statistical significance of the PDs as explanatory variables of default rates.

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4 Duffie and Wang (2004) reach a similar conclusion in their analysis of non-structural credit risk models.

5 The default recovery rate equals the fraction of debt principal that is recovered at the time of default.

1. The models

I examine six structural credit risk models that extend the analytic framework of Merton (1974). That framework focuses on the credit risk of an individual borrower (as opposed to portfolio credit risk), which defaults when its assets fall below a particular threshold. The determination of the threshold differs across the models considered below.

Five of the models are developed in academic articles and, according to the way they tackle the decision to default, can best be divided into two categories. The models in the first category adopt an exogenous default-trigger value of assets. In contrast, the models in the second category derive the decision to default endogenously, as part of the borrower's optimisation problem. Thus, the endogenous default trigger is a function of borrower characteristics.

The sixth model belongs to MKMV and not all of its features are publicly available. From what is known, this model assumes that the default boundary depends on the maturity structure of the debt instruments issued by the obligor.

The five "academic" models assume that the asset value, V_t, evolves as follows:

$$dV_t / V_t = (r_t + \lambda_t - \delta)dt + \sigma dW \tag{1}$$

where r denotes the risk-free rate, λ is the asset risk premium, δ is the asset payout ratio (reflecting, for example, dividend and coupon payments), W is a Wiener process and σ is the instantaneous asset volatility. Some of the models allow the interest rate or the risk premium to be time varying.

Given the process in (1), the PD over a particular horizon equals the probability that the first passage of V below the default trigger, V^*, occurs within that horizon. V^* is a constant in all but one of the five academic models.[6]

In such a setup and for a given initial value of assets, V_0, individual parameters affect the theoretical PD as follows:

$$\frac{\partial PD}{\partial V^*} > 0, \quad \frac{\partial PD}{\partial \sigma} > 0, \quad \frac{\partial PD}{\partial r} < 0, \quad \frac{\partial PD}{\partial \lambda} < 0, \quad \frac{\partial PD}{\partial \delta} > 0 \tag{2}$$

The PD increases in the default trigger, V^*, and in the level of risk, captured by the asset volatility, σ. The implications of the remaining three parameters, which determine the drift in the value of assets, are best considered together. Tight credit conditions, caused by contractionary monetary policy that raises the risk-free rate, r, and/or by stronger aversion to risk that raises the risk premium, λ, seem to counterintuitively depress PDs. At the same time, however, tight credit conditions would tend to raise the payout ratio, δ, which increases PDs.

Some models incorporate a second stochastic process: of the risk-free rate, the asset risk premium or the default boundary. In such models, the PD depends also on parameters that define the second process and its co-movement with asset values. The discussion of these parameters is deferred to the next subsection, which discusses model-specific issues, and to the section on the calibration methodology.

1.1 Models with an exogenous default boundary

In the three "exogenous default" models, the threshold level of assets, V^*, is left unspecified and is typically chosen to be in accordance with aggregate historical data. In particular, when the fraction of assets lost in default is α and the face value of debt is P, V^* is set so that the quantity $\frac{(1-\alpha)V^*}{P}$ equal an estimate of the debt recovery rate after default.[7]

In addition, all three models in this category assume that debt is of infinite maturity. The assumption delivers analytic tractability but makes it impossible to capture the empirical regularity that borrowers are less likely to default over a given horizon if they are to repay the debt principal further in the future.

The three models are distinguished by their choice of a second stochastic process, which complements the process of assets in (1). The second process is introduced in order to allow the setups to capture stylised features of the data that have potentially important implications for credit risk. I outline these implications in the remainder of the subsection.

1.1.1 The model of Longstaff and Schwartz (1995)

In the LS model, the value of assets interacts with a stochastic risk-free rate of return. The correlation between the two random variables is assumed to be negative, which intends to capture the cooling effect of higher interest rates on the macro economy. Specifically, equation (1) is augmented by:

$$dr_t = k_r\left(\bar{r} - r_t\right)dt + \sigma_r dW_t^r \tag{3}$$

where $\bar{r}$ is the long-run risk-free rate of return, k_r reflects the speed of mean reversion, σ_r is the instantaneous volatility of the risk-free rate of return and $corr\left(dW_t^r, dW_t\right) \equiv \sigma_{rv} < 0$.

Under such a specification, a change of r has an ambiguous impact on the PD. By expression (1), a higher interest rate increases the deterministic drift in the value of assets and, ceteris paribus, lowers the PD. Nevertheless, since $\sigma_{rv} < 0$, a higher r tends to be associated with a negative shock to the

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6 Two of the "exogenous boundary" models *assume* that V^* is a constant. In contrast, the "endogenous boundary" models *derive* V^* as a time-invariant function of constant parameters.

7 If the model allows for time variation in V^*, the above procedure would set the initial value of the process.

value of assets, which raises the PD. The relative importance of the latter impact of the risk-free rate increases when asset volatility, σ, is higher and/or the correlation $\frac{\sigma_{rv}}{\sigma_r \sigma}$ is closer to –1.

1.1.2 The model of Collin-Dufresne and Goldstein (2001)

Collin-Dufresne and Goldstein observe that firms tend to issue more (less) debt when their asset value increases (decreases). This leads to mean reversion in the leverage ratio (ie the ratio of debt to assets) – an empirical regularity that the LS model does not account for. The CDG model accommodates the empirical regularity, which implies that the default trigger, V_t^*, moves in step with the value of assets, V_t. Under the maintained assumption that V_t^* is a constant fraction of debt:

$$d\ln V_t^* = k_l\left(\ln V_t - \ln V_t^* - \nu\right)dt \quad , \qquad k_l > 0\,, \;\; \nu > 0 \tag{4}$$

The parameters ν and k_l have direct implications for the theoretical PD. In particular, $\nu > 0$ implies that, in the absence of (transitory) shocks to assets, their value would stay above the default trigger: ie the firm would be inherently solvent. The closer is ν to zero, however, the stronger is the tendency of V_t^* and V_t to converge to a common value. This increases default risk, ie the risk of V_t falling below V_t^*. For its part, a higher k_l implies that the ratio $\ln\left(V_t / V_t^*\right)$ is more likely to stay close to its long-run value ν. Since the latter value is assumed to be positive, an increase in k_l lowers the PD.

1.1.3 The model of Huang and Huang (2003)

There is empirical evidence that equity risk premiums tend to move countercyclically and are, thus, negatively correlated with returns on broad equity indices. On the basis of such evidence, the HH model postulates negative correlation between the risk premium and unexpected shocks to the return on the assets of the typical borrower. Specifically, (1) is augmented by:

$$d\lambda_t = k_\lambda\left(\bar{\lambda} - \lambda_t\right)dt + \sigma_\lambda dW_t^\lambda\,, \qquad corr\left(dW_t^\lambda, dW_t\right) \equiv \sigma_{\lambda v} < 0 \tag{5}$$

A higher $\bar{\lambda}$ implies a higher long-run drift in the value of assets, which, ceteris paribus, lowers the PD. The impact of $\bar{\lambda}$ is stronger the larger is the mean-reversion parameter k_λ. In addition, since $\sigma_{\lambda v} < 0$, a negative value of dW_t, which puts upward pressure on the PD, tends to be counteracted by an increase of the drift in the value of assets.

1.2 Models with an endogenous default boundary

The two "endogenous default" models let a borrower decide when to default. The frameworks differ mainly in the assumptions underlying the default decision. The AST model allows obligors to renege on and then alter the terms of their debt contract. In contrast, a renegotiation is not possible in the LT model, in which borrowers service their debt as long as doing so is justified by the expected future return on equity.

The two models differ also in their assumptions regarding the time to maturity of debt contracts. Namely, the AST framework incorporates perpetual bonds, whereas the LT model assumes that the firm continuously issues debt of a constant but finite time to maturity.

1.2.1 The model of Anderson, Sundaresan and Tychon (1996)

At the time of default, creditors in the AST model can either (i) liquidate the borrowing firm and seize its assets net of a bankruptcy cost or (ii) accept the terms of a new debt contract. Since a liquidation of the borrowing firm is the worst possible outcome for its equity holders, they propose a post-default contract that is acceptable to creditors.

To rule out arbitrage opportunities in this setup, it is necessary that the value of debt increases continuously in the value of assets. In particular, the no-arbitrage condition requires a smooth switch between the pre-default and post-default value of debt. On the one hand, given a fixed bankruptcy cost, K, incurred only if the creditors liquidate the borrower, the post-default value of debt is set by equity holders to equal $V_t - K$. This renders creditors indifferent between re-contracting and liquidating the borrower. On the other hand, the pre-default value of debt is also an increasing function of the firm's assets but is shifted upward by a higher risk-neutral drift in their process (ie, a higher r [8] and / or a lower δ), a higher debt principal, P, a higher coupon rate, c, a lower asset volatility, σ, and a lower monitoring cost, m.[9]

When assets equal the equilibrium default trigger, V_{AST}^*, the post- and pre-default values of debt are the same. A decline in bankruptcy costs K boosts the post-default value of debt, decreases debtors' bargaining power and induces them to wait longer before renegotiating, ie to set a lower V_{AST}^*. In contrast, an upward shift in the pre-default value of debt prompts debtors to negotiate a more advantageous contract earlier: ie set a higher value of V_{AST}^*. In the light of the previous paragraph, this leads to the following comparative statics:

$$\frac{dV_{AST}^*}{dK} > 0,\ \frac{dV_{AST}^*}{dr} > 0,\ \frac{dV_{AST}^*}{d\delta} < 0,\ \frac{dV_{AST}^*}{dP} > 0,\ \frac{dV_{AST}^*}{dc} > 0,\ \frac{dV_{AST}^*}{d\sigma} < 0,\ \frac{dV_{AST}^*}{dm} < 0 \qquad (6)$$

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8 An increase in r also decreases the present value of coupon payments, which lowers, ceteris paribus, the pre-default value of the bond. The data lead, however, to parameterisations of the model, under which the net impact of r reflects the channel specified in the main text.

9 The original AST model does not incorporate monitoring costs. They are introduced here in order to dampen the sensitivity of theoretical PDs to changes in coupon payments, cP. In mathematical terms, the original formula for V_{AST}^* is generalised by replacing cP with $(1-m)cP$. When $m = 0$ the AST model *over*-predicts ex post default rates of BB-rated firms by 3.6 percentage points on average. For BBB-rated and B-rated firms the over-prediction is, respectively, by 1.1 and 4.7 percentage points.

1.2.2 The model of Leland and Toft (1996)

In the LT model, the borrower forfeits its equity value as soon as it does not fulfil a contracted obligation. Thus, the willingness to service debt increases (ie the default trigger V_{LT}^* decreases) in the value of equity, which equals the value of the firm net of the value of its debt.[10] Ceteris paribus, the value of the firm decreases in the default cost, which is assumed to be an exogenous fraction α of assets. In contrast, since it is assessed over the infinite horizon, the value of the firm is insensitive to the time to maturity, T, of the continuously issued debt contracts. For its part, the value of the finitely lived debt decreases in α but by less than the value of the firm. The value of debt decreases also in T, a rise in which heightens the risk of a default before the contract matures. The upshot is that the value of equity (the default trigger V_{LT}^*) decreases (increases) in the default cost but increases (decreases) in the time to debt maturity.

The implications of the other model parameters are similarly rationalised. The coupon rate, c, the principal, P, and the asset payout rate, δ decrease, while the risk-free rate, r, asset volatility, σ, and tax benefits, τ, increase the value of equity. Thus, taking into account the discussion in the previous paragraph, one obtains:

$$\frac{dV_{LT}^*}{d\alpha} > 0,\ \frac{dV_{LT}^*}{dT} < 0,\ \frac{dV_{LT}^*}{dc} > 0,\ \frac{dV_{LT}^*}{dP} > 0,\ \frac{dV_{LT}^*}{d\delta} > 0,\ \frac{dV_{LT}^*}{dr} < 0,\ \frac{dV_{LT}^*}{d\sigma} < 0,\ \frac{dV_{LT}^*}{d\tau} < 0 \qquad (7)$$

1.3 The MKMV model

The sixth model belongs to the commercial service MKMV. A step in the MKMV approach estimates the asset value and asset volatility of the borrowing firm. The step is based on: (i) an option pricing model; (ii) data including equity prices and contractual liabilities; and (iii) information about the borrower's size, industry, profitability and geographical location. Another step of the MKMV approach delivers a default-trigger value of assets, which increases in the borrower's book liabilities. In the determination of the default barrier, short-term liabilities are weighted roughly twice as much as long-term liabilities. In addition, there is an underlying assumption that a default occurs as soon as the lender incurs economic loss.

Finally, an MKMV proprietary model uses the estimates of the borrower's asset value, asset volatility, and default boundary to deliver a firm-specific PD. The model is estimated on the basis of historical default rates and credit spreads. Those data are obtained from the largest available public-firm default database, which has been collected by MKMV.[11]

To the extent that MKMV's rich proprietary data sources have value added and/or the future resembles the past, the commercial service would produce better out-of-sample forecasts of default

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[10] In the LT model, the (market) value of the firm equals the asset value plus the value of tax benefits, less the value of bankruptcy costs, over the infinite horizon.

[11] For a more detailed account of, and references on, the MKMV approach, see Leland (2002).

rates than the academic models, the calibration of which is based exclusively on public data and is not validated in sample. This fact motivates including MKMV PDs in the analysis, even though, as I explain in the next section, they are available in quite short time series.

2. Data: sources, filtering and descriptive statistics

Data availability limits the analysis to corporate borrowers domiciled in the United States. Firm-specific borrower and debt characteristics are provided by Moody's (rating of senior unsecured debt,[12] coupon rate and time to maturity of outstanding bond issues), Bloomberg (book value of total debt[13] and market capitalisation) and Datastream (price of equity and dividend rate). In addition, data on the face value of defaulted debt and its price 30 days after default, provided by Moody's, help estimate default recovery rates.

Moody's also provides data on default rates over different time horizons.[14] The one-year default rate, for example, is calculated as the number of firms that defaulted within a year divided by the number of firms that could have defaulted within that year. In most general terms, Moody's defines default as the instance in which the lender incurs an economic loss, which matches an assumption of all the models considered in this paper.[15] The firms, tracked for the calculation of a particular default rate, are chosen according to the rating of their senior unsecured debt and, in this paper, the focus is on firms with a BBB, BB, or B rating.[16] The average number of firms tracked for the calculation of default rates is 517 (BBB rating), 389 (BB rating), and 482 (B rating). Firms in higher rating classes only rarely fail on debt obligations. Thus, the default history in those rating classes carries little information with respect to changes in the creditworthiness of the constituent firms. Further, Moody's coverage of firms rated C or below is limited and prevents meaningful analysis.

The macroeconomic variables are provided by the IMF, the Congressional Budget Office and the BIS and consist of: an index of US asset prices;[17] the US GDP gap; the US credit-to-GDP ratio; the term spread in the Treasury rate; and the one-year Treasury rate. The first two variables are deflated by the US CPI. The credit-to-GDP ratio and the asset-price index reflect the credit cycle and are used as gaps from their respective stochastic trends. Following Borio and Lowe (2002), the trends are

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[12] When a firm does not have senior unsecured debt, Moody's interpolates the rating.

[13] Total debt includes all interest-bearing obligations.

[14] Overall, the paper uses the Default Risk and Credit Risk Calculator databases of Moody's Investors Service. The databases provide information about all bond issues in Moody's rating universe as well as ratings and default data.

[15] Mood§y's definition of default is spelled out in Moody's Investors Services (1998).

[16] In terms of Moody's rating convention, BBB corresponds to a rating between Baa1 and Baa3, BB to Ba1-Ba3, and B to B1-B3.

[17] The index is a weighted geometric mean of equity prices, residential and non-residential property prices. The weights change through time and are based on households' annual net wealth. The data sources are: S&P Corporate 500 (equity prices); National Council of Real Estate Investment Fiduciaries (commercial property prices) and US Office of Federal Housing Enterprise Oversight (residential property prices).

calculated on the basis of data available in real time.[18] The term spread and the GDP gap reflect developments on the real side of the economy. The former variable is set equal to the difference between the ten-year and three-month Treasury rates, whereas the latter variable is calculated as the difference between log real GDP and log potential real GDP. Finally, the one-year Treasury rate is used for estimating the risk-free rate of return.

The calculation of firm-specific PDs requires data from different sources and it is the intersection of the Moody's and Bloomberg datasets that restricts the sample size. The upshot is that the available data allow for obtaining firm-specific PDs at a quarterly frequency: from Q1 1990 to Q2 2003. The smallest cross sections of PDs are at the beginning of the sample: 16 BBB-, 15 BB- and 6 B-rated firms. The cross-sections then expand monotonically through time and attain an average (maximum) size of: 77 (140) for BBB-, 77 (127) BB-, and 59 (172) for B-rated firms. The sample is dominated by non-financial firms, which constitute 86%, 91% and 92% of, respectively, the "BBB", "BB" and "B" firms.[19, 20]

Besides the PDs implied by structural models from the academic literature, I also examine expected default frequencies (EDFs) calculated by the commercial service MKMV. The EDFs are MKMV estimates of corporate borrowers' one-year default rates and, as such, are the exact counterparts of the one-year PDs implied by the academic models. In comparison to the data employed for the calculation of "academic" PDs, the available MKMV EDFs feature richer cross sections, consisting on average of 319 BBB-, 279 BB- and 277 B-rated firms, but span twice as short a time period, from Q4 1996–Q2 2003.

3. Calibration methodology

In this section, I outline the calibration of the academic models. Except for rare cases, discussed in Sections 1.1.1–1.1.3, the analytical solutions of the models are obtained under the assumption that the parameters stay constant through time. Since key parameters reflect risk premiums, debt-service payments, equity volatility, etc, the assumption is unrealistic and should be interpreted as referring to steady-state borrower characteristics that convey a long-term level of risk. One of the objectives of this paper, however, is to evaluate the *time path* of default rate forecasts that have a short, one-year, horizon. Over such a horizon, the credit risk of a firm depends significantly on transitory shocks to its characteristics. In the light of this, I calibrate the model parameters to their short-term estimates,

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[18] Specifically, a date-*t* point on the trend is calculated via a Hodrick-Prescott filter, which uses data only up to time *t*. The parameter of the HP filter is set to 1600.

[19] Since financial firms enter Moody's calculation of default rates, the consistency of the analysis requires that such firms be considered in the derivation of theoretical PDs. That said, excluding financial firms from the sample leaves the results virtually unchanged.

[20] The reported sample sizes are obtained after filtering the data in order to exclude leverage ratios, dividend rates and equity volatilities that do not belong to the interval (0,1). (The calibration of leverage and equity volatility is described in Section 3). Such a filter removes a relatively small number of observations and is unlikely to influence the analysis. In addition, I also filter out firm-quarter observations that imply a default-trigger value of assets that is larger than 90% of the assets' initial value. A similar but more drastic filter is applied by MKMV: whenever the MKMV model delivers a PD greater than 20%, the PD is reported as equal to 20%.

obtained at the time when a PD is constructed. Provided that debt characteristics are expected to change little over the PD horizon, the calibration procedure is largely consistent with the assumptions of the models and allows them to capture time variability in firms' credit risk.[21]

To the extent that the data permit it and subject to issues of comparability across models (described below), I calibrate the model parameters at the firm level and update them in each quarter of the sample.[22] More specifically, I obtain yearly values for the coupon rate, c, and time to maturity, T, directly from data on bond issues.[23] The maintained assumption is that c and T are representative for all (bank and non-bank debt) of the obligor. Turning to the risk-free rate, r, I estimate it in two alternative ways. First, I consider theoretical predictions based on a constant value of r, which equals the average one-year Treasury rate over the entire sample. Then, I base another set of results on a quarterly time series of r: an entry in that series equals the average Treasury rate in the corresponding quarter.

The initial value of assets, V_0, enters the parameterisation of the models only as a fraction of the default boundary V^*. Since, recalling (1), assets are assumed to follow a geometric Brownian motion, the exact value of V_0 is normalised to 100 without loss of generality.

I set the debt principal $P = l * V_0$ after calculating the leverage, l, as the ratio of the book value of total debt to the sum of total debt and market capitalisation of the firm. In turn, following Huang and Huang (2003), I set the payout ratio $\delta = l * c + (1 - l) * d$, where d is the dividend rate paid out to the firm's equity holders.[24] Both l and d are calibrated quarterly and at the firm level.

I derive firm-specific values of the asset risk premium and volatility at the quarterly frequency by first estimating the corresponding equity premium, λ^e, and equity volatility, σ^e. Since equation (1) implies that the value of equity follows a geometric Brownian motion, I estimate a firm-specific σ^e as the standard deviation of equity returns realised over the year ending with the current quarter. For its part, the estimation of λ^e proceeds in three steps. In the first step, I make use of Tarashev and Tsatsaronis (2005), which employs options data to estimate, inter alia, a time varying risk premium for the S&P 500 stock market index. On the basis of that estimate, I obtain a quarterly time series of market premiums, which peaks in the late 1990s and averages 8%. In the second step, I use results of Bhandari (1988),

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[21] For the "endogenous default" models, one also needs to assume that the parameter variability is small enough to have a negligible importance on the default trigger value of assets. In general, the latter value would depend on both short-term and long-term borrower characteristics.

[22] If a parameter is stochastic in model but fixed in another one, the parameter's initial value in the former model is set equal to its constant value in the latter model. This procedure follows Huang and Huang (2003).

[23] For each firm-year pair in the sample, I set c and T to be equal, respectively, to the average coupon rate and time to maturity of the firm's outstanding bond issues. The averages use weights proportional to the face values of the corresponding bonds. Since, on a firm-by-firm basis, the average time to maturity declines typically through time, the calibrated value of T is roughly twice as large as the average time to maturity over the remaining life of the firm's outstanding debt. This is consistent with the maturity structure assumed in the LT model.

[24] Ideally, d would also incorporate sales and repurchases of equity shares. The employed data sources do not, however, provide a comprehensive coverage of that variable.

which derives a firm-level relationship between leverage and risk premiums. Employing Bhandari's estimate of that relationship and firm-specific leverage ratios, I obtain quarterly cross-sections of firm-specific equity premiums. Finally, I transform each of these cross sections so that their average values match the corresponding stock market risk premiums from the first step.[25]

The conversion of equity premiums and volatility into their asset counterparts, λ and σ, uses theory-implied relationships between the value of equity and the value of assets. In order to ultimately underscore differences in the way the theoretical models process similar information, I focus on two simple (and similar) specifications of equity as a function of assets. One of these specifications is derived within the LS model after setting the volatility of the risk-free rate to zero; the other specification is implied by the LT model in the limit in which the time to maturity of debt shoots to infinity.[26] The first specification is used for estimating λ and σ in the three "exogenous default" models, whereas the second specification is employed for the calibration of the "endogenous default" models.[27]

For the calibration of certain parameters, which are constant across firms and time, I rely on the extant literature and especially on Leland (2002) and Huang and Huang (2003). Namely, for the LS model, I set: $k_r = 0.226$, $\sigma_r = 0.0468$, $\bar{r} = 0.062$, $\sigma_{r,V} = -0.25$; for the HH model: $k_\lambda = 0.202$, $\sigma_\lambda = 0.031$, $\bar{\lambda} = 0.04165$, $\sigma_{\lambda,V} = -0.35$; and for the CDG model: $k_l = 0.2$, and $\nu = 0.7523$.[28] In addition, I adopt $\tau = m = 0.15$.[29]

The last two parameters that remain to be set are: the fraction of assets lost in default, α, (or the fixed bankruptcy cost, K, in the AST model) and the default-trigger value of assets, V^*. The determination of these parameters relies on an estimate of the default recovery rate, which is defined as the price of debt 30 days after default divided by the associated face value. The estimate of the recovery rate, ρ, is allowed to change from year to year but stays constant across firms in each quarter.

I base the estimate ρ on information available to bond holders and obligors in real time. That information is likely to consist of past recovery rates and additional news that is reflected in next-in-line

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25 The transformation is necessitated by the fact that the estimates of Bhandari (1988) do not account for time variation in the market risk premium.

26 The simplified LS and LT setups allow for analytic expressions of asset premium and volatility as functions of equity premium and volatility. Those setups are examined by Huang and Huang (2003) where they are referred to as the "base case" and "endogenous default boundary" models, respectively.

27 Switching between the two specifications has no material impact on the implied PDs.

28 I also explored the implications of alternative parameterisations of the HH and CDG models. In these parameterisations, $\bar{\lambda}$ and ν were calibrated to firm-specific data: the time average of a firm's asset risk premium and leverage ratio, respectively. Switching to these alternative parameterisations has virtually no effect on the implied PDs.

29 Estimating the monitoring costs, m, is beyond the scope of this paper, whereas the adopted value of τ is as assumed in Leland (2002). In the light of the similar implications of m and τ for the PDs implied, respectively, by the AST and LT models, I assign the same value to both parameters.

defaults. In the light of this, I set each year-specific ρ equal to two-thirds times the mean recovery rate of *all* the defaults up to the current calendar year plus one-third times the mean recovery rate in the current year.[30] For consistency with the way Moody's calculates default rates, and with technical assumptions of the theoretical models under study, I calculate recovery rates using only defaults on senior unsecured debt.[31]

The procedure for assigning values to the default trigger, V^*, and the default cost, α (or K), is model specific. Namely, the calibration of V^* reflects the fact that the "endogenous default" and "exogenous default" models treat borrowers' decision to default differently. In addition, the models in the former group allow for deriving the value of α (or K) on the basis of other, independently calibrated, model parameters. By contrast, the "exogenous default" models do not provide any guidance regarding the value of α. Nevertheless, the two types of models incorporate the default costs in a conceptually identical way, via an exogenous constant. This prompts aligning the value of α across models.

For the two "endogenous default" models, the values of V^* and α (or K) are determined *simultaneously* by the requirement that, in each quarter-rating class, the average (latent) recovery rate of the riskier 50% of the firms is to equal the current estimate of the market-wide recovery rate:[32]

$$\rho_t = \sum_{i \in N_t} \frac{(1-\alpha_t) V^*_{LT,it}(\alpha_t)}{P_{it}}, \qquad \rho_t = \sum_{i \in N_t} \frac{V^*_{AST,it}(K_t) - K_t}{P_{it}} \tag{8}$$

where t indexes the quarter and i the firm, N_t is the set of the riskier half of the firms and the principal P_{it} is calculated as described earlier in the section. The default costs are kept constant across firms but vary quarterly, whereas the default triggers vary both quarterly and across firms. The first subscript of V^*_{LT} and V^*_{AST} indicates that the default triggers are also functions of *firm-specific* parameters (recall (6) and (7)).

In the case of the "exogenous default" models, I set $\alpha = 0.40$, which is between the value adopted in Leland (2002) (ie, 0.3) and the average α derived in this paper under the LT model (ie, 0.46). Then, I

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30 Alternative calibration procedures include setting ρ equal to the average recovery rate (i) over the entire sample or (ii) up to the current year. The former alternative fixes ρ through time and thus insulates the analysis from any empirical relationship between PDs and losses given default. Implementing the latter alternative, instead of the procedure proposed in the main text, has a small quantitative impact on the implications of the models.

31 The estimate of the default recovery rate in 1990 is based on 33 defaults, whereas the corresponding value of ρ in 2003 is based on 561 defaults. The average number of defaults, which underlie the time series of ρ, is 161.

32 I consider only the riskier firms because they are expected to actually default and, thus, determine the recovery rates in the data. In the AST model, the value of K increases if, instead, one bases its calibration on the riskiest 25% of the firms. This translates into higher model-implied PDs: the average PD of B-rated firms increases from 4.5% to 4.7%. For BB-rated firms, the increase is from 1.2% to 1.9%, and for BBB-rated firms from 0.2% to 0.4%. The impact is similar within the LT model but is attained on the back of unrealistically high values of α. In contrast, the results do not change materially if one calibrates α and K on the basis of all the firms in the cross section as opposed to the riskiest 50%.

allow the default trigger to vary quarterly and across firms by setting it according to the following version of equation (8):[33]

$$\rho_t = \frac{(1-\alpha)V_{it}^*}{P_{it}} \tag{9}$$

Tables 1–3 allow for an appreciation of the calibration methodology and its implications. Table 1 recapitulates all the model parameters and features of their calibration, whereas Table 2 catalogues how changes in the parameters affect theory implied PDs. For its part, Table 3 reports parameter averages alongside characteristics of the representative firm, as adopted by Leland (2002) and/or Huang and Huang (2003).

The differences between the calibration results of this paper and those of the previous literature are due primarily to three factors. First, this paper calibrates separately three rating classes whereas the earlier papers base certain aggregate values on the entire spectrum of ratings. Second, the data underlying the fourth column in Table 3 uses data starting as early as 1970 whereas the data used herein start in 1990.[34] Third, this paper calculates the leverage ratio on the basis of total debt, whereas the earlier papers rely on estimates of leverage that are provided by Standard & Poor's (1999) and incorporate total *liabilities*.[35]

4. The model-implied PDs

Leland (2002) finds that the LT and (a simplified version of) the LS models imply one-year PDs that underpredict consistently and substantially ex post default rates. He conjectures that this might be due to his calibration of the models, which focuses on the "representative" firm endowed with the average borrower characteristics. In examining the cross-sectional variability of firm-specific PDs, this section demonstrates inter alia that Leland's conjecture is indeed borne out.

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33 There is no consensus in the literature regarding the value of α: at one extreme, Andrade and Kaplan (1998) argue that it should not exceed 20%; at the other extreme Leland and Toft (1996) set it to 50%. Reducing the value of α to 30% in the "exogenous default" models does not affect the time pattern of the cross-sectional averages of PDs but shifts them down substantially: by 0.068 percentage points (on average) for BBB-rated firms, by 0.61 percentage points for BB-rated firms and by 2.5 percentage points for B-rated firms.

34 This affects especially the estimate of the risk-free rate of return because the 1990s witnessed levels of the interest rate that were low by historical standards.

35 The leverage used in previous studies is larger because total liabilities provide a broader measure of financial obligations. In addition to debt, total liabilities include obligations that do not involve interest payments (eg promises for physical deliveries). There are three reasons for choosing to work with total debt as opposed to total liabilities. First, a leverage ratio that is based on total liabilities would lead to an overestimation of coupon payments and payout rates. In turn, this would lead to too high values of model-based PDs. Second, total debt tracks more closely the bond instruments underlying the calculation of default recovery rates. Third, total debt underlies the calculation of MKMV PDs, which are used here to benchmark the performance of the academic models. Finally, it is necessary to also acknowledge that, by ignoring a portion of liabilities, total debt is likely to impute too big of a fraction of the equity risk premium and volatility onto the asset risk premium and volatility. The impact of leverage on the calibration of the latter two parameters translates, however, into a small quantitative impact on model-implied PDs.

Irrespective of which set of parameters one chooses to work with in Table 3, the implied one-year probabilities of default are orders of magnitude smaller than the corresponding default rate. This is illustrated by Table 4, which reports average ex post default rates of B, BB and BBB-rated firms together with two sets of PDs: one associated with the representative firm in Leland (2002) and one with the "average" firms in this paper.

Even though the theoretical one-year PD of the representative firm underestimates severely default risk, the models perform starkly better when employed for the calculation of firm-specific PDs. Focusing on one rating class-quarter at a time and averaging the PDs in the associated cross section produces the time series of theoretical predictions portrayed in Figure 1.[36] In the figure, an average one-year PD is aligned with the default rate realised over the following year within the corresponding rating class. Considered even casually, the figure suggests that the bias in average firm-specific PDs vis-à-vis ex post default rates is either negligible or much smaller than the bias in the PDs of the representative or "average" firms.

Table 4 reveals directly the small bias in theoretical default predictions. It is illustrated by the small difference between the time averages of the LT forecasts portrayed in Figure 1 and the corresponding average default rates.[37] Such a small difference is expected to prevail if a valid credit-risk model is applied to a random selection of firms in a rating class and the sample period is sufficiently long.

Figure 1 includes PDs implied by the "endogenous default" setups, the LT and AST models, and only one of the "exogenous default" setups, the HH model. The reason for not showing the implications of the LS model is that they are virtually identical to their counterparts in the HH model. This is due to the two models differing only in their choice of a second stochastic process (for the risk-free rate of return or the risk premium) whose quantitative implication for PDs turns out to be negligible. In order to avoid repetition, I suppress the LS setup from the subsequent analysis. In addition, the CDG model under-predicts consistently the ex post default rates. The result is driven by the two parameters k_l and ν, whose values imply that the default boundary tends to stay far below the value of assets (recall equation (4) and the accompanying discussion). This depresses the theoretical PDs. Having identified a pronounced bias in the implications of the CDG model, I do not include it further in the analysis.[38]

4.1 Firm-level data and theoretical predictions of default rates[39]

In this subsection, I explain the pronounced difference between default-rate predictions based on firm-specific PDs and alternative predictions based on "average" borrower characteristics. To illustrate the

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[36] The series in Figures 1–3 are based on the time-varying calibration of the risk free rate, r. The implications of fixing the value of r through time are discussed in Section 4.5.

[37] The message of Table 4 changes little in the context of the AST, HH and LS models: see Tarashev (2005).

[38] As the speed of mean reversion, k_l, decreases towards zero, the PDs generated by the CDG model converge to the ones generated by the LS and HH models.

[39] For expositional purposes, I focus predominantly on BB-rated firms for the remainder of Section 4. The predictions of the models as regards the default rates of BBB- and B-rated firms can be dissected similarly.

issues involved, I focus on a cross-section of one-year PDs that is derived within the LT setup and is associated with BB-rated firms in Q4 2001.

Figure 2 provides a histogram of the cross-section, which is characteristic of the entire sample and reveals that only a fraction of the firms represent non-negligible credit risk. These firms are behind the long right tail (right skew) of the distribution. Their relative number in the cross section and the magnitudes of their PDs drive the models' predictions of the one-year default rate.

The skew of the distribution is an illustration of the so-called "Jensen inequality" effect, which arises because of a non-linear (convex) relationship between the parameters of the model and the implied PD. The importance of the "Jensen inequality" effect is best appreciated when the PD is considered as a function of only one variable. Figure 3 illustrates such a hypothetical scenario by focusing on the leverage ratios of the firms behind the plot in Figure 2. Figure 3 reveals that the LT model (crystallising in the middle panel) translates a slight skew in the leverage ratios into a highly skewed distribution of PDs when all the other parameters of the model are kept at their cross-sectional averages.

The upshot of the Jensen inequality effect is that the average PD in a cross-section is substantially larger than the PD of the corresponding representative firm, which is endowed with the average borrower characteristics. In the above example, the "average" BB-rated firm in Q4 2001 has a model-implied PD equal to 0.18%, whereas the average of firm-specific PDs is 3.7%.

4.2 Time variability of PDs

Figure 1 reveals substantial time variation in the theory-based PDs, which can be rationalised on the basis of the discussion of the models in Section 1 and their calibration outlined in Section 3. To illustrate, I consider here the implications of the LT model for firms rated BB in Q4 2001 and Q1 2002. Between these two quarters, there is a substantial drop of the average PD: from 3.7% to 1.1%.

Zooming on the riskiest 20 firms in each cross-section of firm-specific PDs, ie the firms that drive the default forecasts of the model, the average coupon rate and time to maturity of outstanding debt are virtually identical in the two quarters. In contrast, the average asset pay-out rate, δ, decreases from 6% to 5% while the average leverage ratio, l, decreases sharply from 62% to 49% and the average asset volatility, σ, shoots up from 29% to 37%.[40] Expression (7) indicates that such changes in δ, l and σ decrease the default boundary, V^*_{LT} and, by expression (2), decrease the probability of default. As also indicated by expression (2), the parameters δ and σ affect the PD via a second channel as well: via their implications for the process of assets. It turns out, however, that the first channel dominates.

[40] The change in parameters may seem too abrupt since it occurs between two consecutive quarters. Note, however, that it is due partly to exit/entry of firms from/in the BB-rating class and partly to changes in firm characteristics between the two quarters. In particular, roughly half of the riskiest 20% of the "BB" firms in Q4 2001 are no longer in the BB-rating class in Q1 2002: this contributes to the decrease in leverage ratios reported above. In addition, those of the riskiest BB-rated firms, which stay in the same class over the two quarters, experience on average a 6 percentage point decline in leverage.

Expressions (1)–(9) and the logic of the previous paragraph help rationalise intertemporal changes in the default predictions of each one of the considered theoretical setups. Nevertheless, the understanding of these setups is greatly enhanced by an across-models comparison of the implied PDs.

4.3 A comparison between the "exogenous default" and "endogenous default" models

In this subsection, I identify key factors behind differences in the default-rate forecasts of models in the "exogenous default" group, on the one hand, and those in the "endogenous default" group, on the other. As revealed by Figure 1, these differences are substantial, especially during the second half of the sample period.

The differences between the implications of the two groups of models turn out to be due to disparities in the relative importance of the default recovery rate, ρ, which is estimated on a market-wide basis, and firm-specific leverage, l. The greater the relative role of the former parameter, the smaller is the *effective* dispersion of borrower characteristics in the cross sections and the weaker is the "Jensen inequality" effect described in Section 4.1. In the "endogenous default" LT and AST models, l is the dominant factor behind default-rate predictions because it affects the calibration of the default trigger (and, by expression (2), the PD) via two mutually reinforcing channels. On the one hand, by expressions (6) and (7), V^*_{AST} and V^*_{LT} are increasing functions of the debt principal P and thus l. On the other hand, by equation (8) and for a given default recovery rate, a higher l (P) has to be matched by higher values of V^*_{LT} and V^*_{AST}.[41] In contrast, recalling the discussion in Section 3, only the second channel operates when the default trigger is exogenous, as is the case in the HH model. This weakens the impact of leverage on V^*_{HH}, and on the associated PD, relative to the impact of the default recovery rate, conveyed by equation (9).[42]

A concrete illustration of the argument is obtained by focusing on Q2 1999 and Q4 2000 and considering alternative theoretical predictions of one-year default rates in the universe of BB-rated firms. Over the seven-quarter period, the estimated recovery rate falls from 46% to 37% and the leverage ratio of the riskiest 20% of the firms rises from 65% to 72%, while the other borrower and debt characteristics remain roughly constant. In accord with the above discussion, the change in leverage is at the root of the increase in the average PDs implied by the LT and AST models: from 0.8% to 2.6% and from 0.7% to 3.6%, respectively. In contrast, the change in the recovery rate forces

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41 In mathematical terms, both channels operate because one solves simultaneously for V^*_{LT} and α, in the LT model, and for V^*_{AST} and K, in the AST model.

42 To a lesser extent, other borrower-specific characteristics also explain the differences between the PDs implied by the two groups of models. Time to maturity of outstanding debt, asset payout ratio and asset volatility influence the dynamics of V^*_{LT} and / or V^*_{AST} but none of them enters the calibration of V^*_{HH}.

the calibrated values of V_{HH}^{*} to drop, which decreases the average PD implied by the HH model: from 0.6% to 0.2%.

The ex post default rates of BB-rated firms increases from 1.1% in Q2 1999 to 2.3% in Q4 2000. The above example thus helps illustrate a general finding that, unlike “exogenous default” models, their “endogenous default” brethren are able to capture an increase in the default rate accompanied by a decline in the default recovery rate. The tendency of default and recovery rates to move in opposite directions has been recorded in the literature as an empirical regularity.[43]

4.4 A comparison between the “endogenous default” models

Figure 1 also helps detect time periods, over which the predictions of the two “endogenous default” models move in opposite directions. By expressions (2), (6) and (7), this phenomenon must be due to a change in: (i) the asset payout ratio, δ, a rise in which raises V_{LT}^{*} but lowers V_{AST}^{*}; and/or (ii) the risk-free rate, r, a rise in which lowers V_{LT}^{*} but raises V_{AST}^{*}.

The predictions of the two models regarding BB-rated firms in Q3 1992 and Q4 1994 provide a case in point. Between the two quarters, the riskiest 20% of the firms in the cross-sections exhibit a drop in the asset payout ratio (from 8% to 7% on average) while the risk-free rate shoots up (from 3.4% to 6.6%). The changes in these two parameters drive the cumulative changes in the theoretical default boundaries over the seven-quarter period and lead to a 2.5 percentage point *de*crease in the average PD implied by the LT model but a corresponding 1.5 percentage point *in*crease within the AST model. The story reverses between Q1 2001 and Q1 2003 when a drop of the risk-free rate from 4.6% to 1.3% results in the LT (AST) model pointing to an increased (decreased) default risk.

4.5 The impact of different calibrations of the risk free rate

It has been documented in the literature that Treasury yields, which enter the determination of the PDs portrayed in Figure 1, might lead to a poor estimate of the risk-free rate of return, r.[44] To examine the sensitivity of the structural credit risk models to the value of r, I shut off its time variability by setting it equal to the average of the one-year Treasury rates over the entire sample.

The set of model-implied PDs, obtained under the time invariant estimate of r, is portrayed in Figure 1a along with the corresponding ex post default rates. In comparison to the plots in Figure 1, the overall level of the PDs remains largely unaffected but they tend to match more closely the actual default rates. The closer match is more pronounced in the context of the “endogenous default” models, whereas changes in the estimate of the risk free rate have a limited impact on the time profiles of “exogenous default” PDs.

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[43] See Altman et al. (2003) and Altman et al. (2004).

[44] See, for example, Feldhütter and Lando (2004).

Recalling Section 3, the calibration of the exogenous default trigger is independent of the risk-free rate, which enters the determination of "HH" PDs *only* via the law of motion of assets (equation (1)). Since that law of motion is the same across all academic models, the "HH" lines in Figures 1 and 1a convey a general implication: the impact of the different calibrations of r on the drift in asset values tends to be translated into a limited impact on PDs.[45]

For their part, the endogenous default triggers V^*_{AST} and V^*_{LT} do depend on the risk-free rate. As a result, setting r to be constant through time smoothes out swings in the PDs implied by the AST and LT models. Since the risk free rate affects the two default-trigger values in opposite directions (it raises V^*_{AST} but lowers V^*_{LT}) the move from a time-varying to a constant r also increases the correlation between the PD series delivered by the two "endogenous default" models.

4.6 Theory-based predictions of default rates over longer horizons

From the point of view of lending institutions, the relevant horizon of a PD reflects the time period needed for the disposal of credit risk and the remaining life of the particular debt instrument. Risk-management considerations should thus be expected to often draw banks' attention to PDs with horizons beyond the one-year one considered in the paper so far.

Figure 4 compares five-year ex post default rates to the corresponding PDs implied by the LT, AST and HH models. The calibration of the models underlying Figure 4 is at the yearly frequency and, in most general terms, adopts the methodology outlined in Section 3 and used for the calculation of one-year PDs. However, I implement the notion that the long-term (steady-state) characteristics of a firm become more relevant as the horizon of the PD increases. To proxy for the steady state at the borrower level, I set the firm-specific parameters, the default recovery rate and the risk-free rate of return equal to their time averages.[46]

Several lessons can be drawn from Figure 4. Against the backdrop of one-year PDs, the main message is that the bias in the theoretical predictions of default rates remains small or virtually non-existent over longer horizons as well. Furthermore, in all three rating classes, the theory-implied PDs capture the increase in ex post default rates from 1995 to 1999. In contrast, the time pattern of defaults in the first half of the sample is matched only partially at best.

A comparison of Figure 4 to Figures 1 or 1a reveals that the differences across models are smaller when the horizon of default forecasts is longer. On the one hand, this is driven by the underlying calibration approaches. The one used for five-year PDs smoothes the characteristics of each rating class across time and thus significantly dampens the transient swings in model-implied PDs. On the

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[45] An exception to this general finding occurs in 1990–91. When the risk-free rate is allowed to change through time, the high interest rates in these two years lead to a large positive drift in the asset value and depress substantially the PDs delivered by the HH model.

[46] Averaging firm-specific parameters through time has a limited impact on the overall level of theoretical PDs but reduces their intertemporal variability. Specifically, the average PD in a rating class changes through time only as a result of the exit (entry) of firms out of (in) that rating class.

other hand, the finding is also due to the three models differing (to a first-order approximation) only in their determination of the default-trigger value of assets. As the horizon increases, the uncertainty about future asset values intensifies and theoretical PDs become less sensitive to changes in the default trigger.

Mimicking the construction of Table 4, Table 5 helps appreciate the bias in alternative model-based five-year PDs vis-à-vis the corresponding default rates. A comparison of the last two columns in the table reveals that the "Jensen inequality" effect, discussed at length in Section 4.1, is still at work. Namely, the average firm-specific PDs are significantly larger (and closer to the ex post default rates) than the PDs of the "average" firm. [47]

5. Economic significance of the forecast errors in model-implied PDs

The analysis in Section 4 helps one rationalise the performance of the different credit risk models. For example, the evolution of firms' leverage is seen to drive the close match between the one-year "BB" PDs implied by the "endogenous default" models and the corresponding default rates.[48] For its part, the over-reliance of the calibration of the "exogenous default" models on the default recovery rate results in persistent underprediction of one-year default rates across all rating classes. In the context of B-rated firms, markedly low leverage and asset payout ratios lead all models to severely under-predict the default rates in 1990 and 1991.

Having rationalised the implications of the academic models, I evaluate them in economic terms in this section. In particular, I consider a lender who determines the amount of capital to set aside by assessing the credit risk of an exposure on the basis of one of these models. Then, I compare the lender's capital to two benchmarks: (i) capital based on perfect knowledge of credit risk; and (ii) capital based on the credit rating of the exposure. Except in the latter benchmark case, I treat PDs as the (sole) measure of credit risk and assume that the lender adopts the foundation IRB approach of Basel II. That approach delivers regulatory capital solely on the basis of a PD estimate.

The short time span of the MKMV PDs limits significantly the scope of their evaluation. Nevertheless, by being based on a particularly rich proprietary dataset, these PDs provide an additional perspective on the implications of the academic models.[49]

Consider six hypothetical banks that invest in a bond, the *ex ante* PD of which equals the *ex post* default rate in a particular rating class (BBB, BB, or B). Two of the banks are "benchmark" ones. One

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[47] Table 5 suggests that, as the rating of the firms improves, so does the performance of the Leland (2002) parameterisation of the LT model. This, however, results from Leland's calibrating virtually all of the parameters (the exception is leverage) to borrower characteristics averaged across all rating classes. Such a calibration inflates artificially the riskiness of BBB-rated firms and understates the riskiness of B-rated firms.

[48] Refer to Figures 1-3 or 1a–3a.

[49] The data cross sections underlying the PDs of the academic models are, in general, different from the cross sections underlying the MKMV PDs. The working assumption is that each cross section is representative of the cross section determining the corresponding ex post default rate.

of them is assumed to possess perfect foresight regarding the probability that the bond defaults over the next year. To calculate the regulatory capital of that bank in a particular quarter, I use the *ex post* one-year default rate in the relevant rating class and the foundation IRB approach. The other "benchmark" bank adopts the standardised approach of Basel II and, thus, calculates its capital requirements on the basis of the rating of the bond issuer: the capital requirements of this bank do not change through time. Each one of the remaining four banks relies on a particular model: the LT, AST, HH or MKMV model. In each quarter of the sample, such a bank determines its regulatory capital on the basis of the average one-year PD implied by the adopted model and the foundation IRB approach.[50]

In order to be able to draw sharp conclusions, I assume that the optimal capital levels are those calculated by the "perfect foresight" bank.[51] At the other extreme is the bank adopting the standardised approach (SA). That approach relies on publicly available credit ratings which, in principle, reflect borrowers' *relative* creditworthiness as opposed to probabilities of default.[52] Thus, in the context of this section, a credit risk model provides value added only if the bank relying on it matches the optimal level of capital more closely than the "SA" bank.[53]

Table 6 summarises the results of the exercise. The table is divided into three vertical panels, each one of which corresponds to a particular rating class. The top rows provide descriptive statistics of the optimal capital requirements. The bottom rows report mean and mean absolute discrepancies (or errors) between the optimal capital requirements and those deduced by the "LT", "AST", "HH", "MKMV" and "SA" banks. The mean error reveals whether the small bias in theoretical PDs vis-à-vis default rates translates into a small bias in model-implied capital. For its part, the mean absolute error reveals whether the model-implied and optimal capital levels tend to stay close to each other through time. In general, the sample period is from Q1 1990 to Q2 2003. The numbers in parentheses are based on the shorter period covered by the MKMV sample: from Q4 1996 to Q2 2003.

Given the higher incidence of default among lower-grade obligors, the regulatory capital of the "perfect foresight" bank increases when moving from BBB to BB and then to single-B rated entities. In addition, the IRB approach maps PDs into capital requirements via a concave function, which dampens (amplifies) the variability of regulatory capital at high (low) PD levels. As a result, the volatility of the

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50 The risk-free rate of return, which is a parameter in the LT, AST and HH models, is assumed to be time invariant for this exercise. In other words, the banks relying on these three models use the PD series portrayed in Figure 1a.

51 Since the exercise is extremely stylised, I make no distinction between regulatory (or required) capital and economic (or optimal) capital.

52 The officially announced objective of credit rating agencies is to rate firms according to their long-term financial characteristics. This means that, conditional on the latter characteristics, credit ratings should not vary across the business cycle, even though PDs are largely procyclical. In addition, credit ratings are supposed to distinguish riskier from safer firms and, thus, need to provide an ordinal ranking of firms but need not reflect an absolute measure of default risk. For further discussion on the issue, refer to Amato and Furfine (2003).

53 Note that the "optimal" capital is based on ex post default rates and thus may incorporate more information about PDs than what could possibly be known ex ante. In this sense, the model-implied capital cannot be expected to match exactly the "optimal" one.

capital requirements of the "perfect foresight" bank stays virtually constant across rating classes, even though the volatility of default rates increases as the credit rating worsens.

The performance of the "SA" bank is mixed. On the one hand, that bank matches quite closely the optimal regulatory capital for single-B exposures. This might reflect an intended feature of Basel II: to have the standardised and the foundation IRB approaches calibrated so that they produce similar capital requirements on the basis of historical default rates of B-rated obligors. On the other hand, the "SA" bank performs significantly worse in the BBB rating class. The latter result reflects the well-known fact that the standardised approach assigns the same regulatory capital (8% per unit of exposure) to both BBB and BB rated exposures. The requirements are conservative and overshoot somewhat the optimal regulatory capital even for the "BB" bonds. In the context of "BBB" bonds, however, the average overshooting is substantial: at 5.3 percentage points, it is *twice* as large as the corresponding mean capital requirement of the "perfect foresight" bank.

The bottom rows of Table 6 suggest that the performance of the credit risk models is quite stable across rating classes. In other words, as the credit rating of obligors deteriorates, the regulatory capital of the "LT", "AST" and "HH" banks increases (on average) by roughly as much as the optimal regulatory capital. Being responsive to changes in the underlying credit risk, the regulatory capital of these four banks improves on the performance of the "SA" bank in the context of BB and, especially, BBB rated obligors. By contrast, there is no such improvement in the context of the riskiest group of borrowers, as foreshadowed by the weak relationship between theoretical PDs and default rates in the B-rating class (bottom panel of Figure 1a).

A comparison across the three "academic" models reveals that, according to the adopted criteria, the LT model provides the best forecasts of ex post default rates. The result holds true for all three rating classes considered and is expressed by the "LT" bank incurring the lowest mean *and* mean absolute errors. The virtual lack of bias in that bank's capital, expressed by mean errors that are a small fraction of the average optimal capital, is particularly impressive. Having said that, the LT model leads to non-negligible point-in-time discrepancies in regulatory capital: in the context of B-rated firms, for example, the mean absolute error is only slightly lower than the standard deviation of the optimal capital requirements. Recalling the bottom panel of Figure 1a, this is due primarily to time periods at the beginning and the end of the sample. As argued by Kurbat and Korablev (2002), however, large point-in-time discrepancies between model forecasts and ex post default rates need not be due to invalid theoretical firm-specific PDs but might be a consequence of a large dispersion in the statistical distribution underlying default rates.[54]

Among the "academic" models, the HH setup leads to the worst match of optimal capital requirements. In addition, the model underperforms the standardised approach within both the BB- and B-rating

[54] The dispersion in the distribution of default rates is more likely to be pronounced the more strongly correlated are individual defaults.

classes. Recalling Figure 1a, the weak performance of the HH model reflects its undershooting ex post default rates over most of the sample.

Surprisingly, the “MKMV” bank does not outperform all the banks relying on the “academic” credit risk models. Across all rating classes, the MKMV model fares consistently better than the HH model in terms of both mean and mean absolute errors. In contrast, the capital requirements implied by the model of the commercial service show an unambiguous improvement upon the LT and AST frameworks only in the context of B-rated firms.

The performance of the MKMV PDs reflects a sustained overprediction of default rates. Known to be based partly on historical data, the parameterisation of the MKMV model may have been influenced unduly at the end of the 1990s by the high levels of credit risk at the beginning of that decade. In addition, however, the short available time series of MKMV PDs are likely to provide a distorted picture of the bias in the model’s predictions. Since, as argued by Kurbat and Korablev (2002), ex post default rates are typically draws from a dispersed right-skewed distribution, their average level would tend to converge to the average of the true firm-specific PD’s only over a long time period.[55]

6. Model-implied PDs and turning points in the outlook of credit risk

The analysis of Section 5 examines time aggregates of the forecast errors of structural credit risk models. The section thus sheds little light on the degree to which theoretical PDs explain the intertemporal evolution of ex post default rates. In order to further the analysis on that front, I incorporate the implications of the models in time-series regressions and report the estimation results in Tables 7, 8 and 9.

In most general terms, I regress one-year ex post default rates on ex ante theory-implied PDs. In all regressions, period-*t* PDs forecast the default rate realised over the year *starting* in *t*. To examine the robustness of the models’ explanatory power, I consider several control variables: the past default rate (realised over the year *ending* in *t*) and macroeconomic indicators (realised *prior* to the year ending in *t)*. The PDs delivered by the ideal credit risk model would incorporate all the currently available information that is useful for forecasting default rates. Such PDs would thus be the sole significant explanatory variable in the regressions.

All regressions include 54 quarterly observations, from Q1 1990 to Q2 2003, and focus on one rating class at a time.[56] The exercise uses the model-implied PD series portrayed in Figure 1a:[57] the series

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55 The length of the time period, over which the bias in a model’s default rate predictions can be meaningfully tested, would depend on the true default risk of the firms in focus, their number and the correlation of their defaults.

56 I report a regression with the lagged dependent variable only when the associated coefficient is statistically significant at the 5% level. Owing to the overlapping horizons of PDs and default rates (recall that I consider quarterly series while the horizon is one year), the regression errors should be expected to be serially correlated. To account for this, I base the derived p values on Newey-West robust covariance matrices (for the regressions pertaining to BB- and B-rated firms) or on Huber-White robust covariance matrices (for the regressions pertaining to BBB-rated firms).

are thus based on the time invariant estimate of the risk-free rate and an entry in them is equal to the average firm-specific PD in a particular quarter. The regressions are weighted with the weight in each quarter increasing in the number of firms that underlie the average theoretical PD in the quarter. Within the BBB-rating class, a significant number of the ex post default rates equal zero and I use a censored-regression specification based on the Tobit model.[58]

6.1 "Models only" regressions

The first four columns in Tables 7, 8 and 9 contain the estimated coefficients of regressions that incorporate exclusively the LT, AST and/or HH models. The first three regressions focus on one of the models at a time, whereas the fourth one represents a "horse race" among all three models.

In accordance with the conclusions drawn from the "regulatory capital" exercise in Section 5, the LT model outperforms the other two structural frameworks in forecasting ex post default rates. The coefficients of the LT PDs are invariably of the expected positive sign and are statistically significant in all three "horse race" regressions and in two out of the three applicable "one model" regressions. In addition, the latter two regressions (which pertain to BB- and BBB-rated firms) score better in the goodness of fit measure than the corresponding regressions incorporating only the AST or HH PDs.

Including all three model-implied PDs as explanatory variables substantially improves the fit of the regressions. The improvement is greatest in the context of the BB rating class where the LT and HH PDs complement each other in capturing the three phases of credit risk observed in the sample: (i) the spike in default rates in 1990–91; (ii) their subsequent drop until 1998; and (iii) the moderate pickup thereafter. On the one hand, the LT PDs match on average the ex post default rates but miss their relative levels in phases (i) and (iii); on the other hand, the HH PDs underpredict over most of the sample but exhibit a global peak in the early 1990s, just like default rates. At another extreme, the models perform worst in forecasting the credit risk of B-rated borrowers. The regression results reflect all the models missing the spike in "single-B" default rates in the early nineties and the sharp downturn starting at the end of 2001.

The AST PD enters insignificantly all "one model" regressions but, surprisingly, attains a statistically *significant negative* coefficient in the "horse race" regressions using data on BBB- and B-rated obligors. Any difference across models with respect to the signs of the associated regression coefficients is due to the asset payout ratio δ, which is the sole parameter with opposite impacts on the LT and HH PDs, on the one hand, and AST PDs, on the other.[59, 60] Thus, the negative regression

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[57] As could be conjectured by comparing Figures 1–3 to Figures 1a–3a, the time-varying estimates of the risk-free rate weaken the explanatory power of the models.

[58] 22 of the 54 ex post default rates in the BBB-rating class equal zero. The zero value of default rates do not imply absence of default risk but are an artefact of there being a finite number of low-risk borrowers. To account for this, I assume that default rates are "censored" at a low positive value (specifically, 0.03%). Under the censored regression model, the reported adjusted R^2 reflects the goodness of fit vis-à-vis the latent dependent variable, which is a linear function of the regressors.

[59] The "horse race" regressions estimate what the relationship between each one of the PDs and ex post default rates would be if the other PDs were to stay constant. Thus, in switching from "one model" to "horse race" regressions, one obtains PD coefficients that reflect to a smaller (greater) degree parameters with similar (disparate) implications in the different models.

coefficient of the AST PD is an empirical rejection of the AST model's implication that default risk decreases in the asset payout ratio.

6.2 Macroeconomic variables and credit risk

As indicated in the previous section and illustrated in Figures 1 and 1a, the theory-based PDs miss the direction of ex post default rates over certain time periods. In the light of the firm-specific calibration of the models, the finding suggests that there might be market-wide determinants of credit risk that are not captured by the models. In order to investigate the presence and importance of such global factors, I incorporate macroeconomic variables in the regression analysis.

In choosing the specific variables, I rely on the extant literature which has identified predictors of turning points in the credit and business cycles. In particular, Borio and Lowe (2002) discover that positive deviations of real asset prices and the credit-to-GDP ratio from their respective trends (estimated on the basis of historical data available in real time) reflect the build-up of financial market imbalances and forecast well banking system distress years down the road. To the extent that the distress is associated with a deterioration of banks' lending portfolios, the two financial variables should help predict spikes in default rates. In addition, in order to account for real-side imbalances that might translate into loosening or tightening of lending criteria, I consider the deviation of real GDP from its potential level as a harbinger of future changes in credit risk. Finally, I also incorporate the Treasury term spread which helps predict changes in real economic activity according to Estrella and Hardouvelis (1991) and Smets and Tsatsaronis (1997). In so far as the business cycle affects firms' willingness/capacity to service their debt, the term spread should also help predict default rates.

With the above motivation, I use the Treasury term spread, the asset-price, credit-to-GDP and GDP gaps, together with model-based PDs, as explanatory variables of default rates. The results are reported in columns 5 to 11 in Tables 7, 8 and 9 and shed light on two related questions: (i) Could the forecast errors of theoretical PDs be attributed to factors related to the business and/or credit cycles? and (ii) Is the explanatory power of theoretical PDs robust to controlling for variables that capture market-wide phenomena?

Focusing on the two financial-side variables, the results suggest an affirmative answer to the first question. The credit-to-GDP gap and the real asset-price gap enter the regressions with statistically significant coefficients and improve substantially the fit.[61] In the context of BB-rated firms, for example, the adjusted R^2 reaches 79%, which is 24 percentage points higher than the highest R^2 in the "models only" regressions. In addition, in line with the conclusions of Borio and Lowe (2002),

[60] Recall Table 2 and that the risk-free rate is being kept constant. The asset payout ratio increases the AST PD via its impact on the process of assets but decreases the AST PD via its impact on the default trigger. The second channel dominates.

[61] When I introduce a single macroeconomic variable in a regression, I pick the lag of the variable that minimises the Akaike information criterion. The lag is allowed to change with the rating class. Later, when two or more macro variables are used simultaneously in a regression, I preserve their initially determined lags.

positive shocks to the financial-side variables indicate a build-up of financial vulnerabilities and tend to increase default rates 3–5 years in the future.[62]

Examination of the real-side macro variables also suggests that the forecast errors of the structural credit-risk models can be attributed in part by factors related to the business cycle. The GDP gap and the term spread improve substantially the goodness of fit of the regressions: in the context of BB-rated firms, for example, the adjusted R^2 reaches 69%. Positive shocks to the GDP gap indicate an overheating of the economy, which tends to be associated with an increase of default rates 1–2 years into the future. In contrast, negative shocks to the term spread are associated with higher default rates within the following 2 years. This is consistent with findings of Estrella and Hardouvelis (1991) and Smets and Tsatsaronis (1997) that a tightening of the term spread heralds recessions up to 2 years in the future.

As regards the second question posed above, the explanatory power of theoretical PDs is largely robust to controlling for financial-side macro variables. The point is seen most sharply when one focuses on the LT PDs whose coefficient remains positive and significant across all rating classes. Except for the BBB-rating class, the same is true for the PDs implied by the HH model. For the reasons discussed in Section 6.1, the AST PDs attain significant negative coefficients.

The inclusion of the GDP gap and the term spread as explanatory variables of default rates suggests that the predictions of the structural credit-risk models may be influenced by market-wide factors. When one forecasts the default rates of BBB- and B-rated firms, the real-side macro variables tend to substitute for the information in theory-based PDs. In the context of BB-rated firms, however, the explanatory power of theoretical PDs is quite robust and *complements* the explanatory power of the GDP gap and the term spread.

For completeness, in column 11 of Tables 7, 8 and 9, I report the results of regressing ex post default rates on *all* the explanatory variables mentioned above. The message of this specification is consistent with the conclusions of the smaller regressions. In addition, one observes that the real-side and financial-side macro variables often fail to provide complementary information regarding default rates.

6.3 MKMV PDs as predictors of default rates

The PDs provided by MKMV explain well the time profile of default rates. These PDs enter regressions of default rates with statistically significant positive coefficients and render the coefficients of other model-implied PDs insignificant and/or negative. The result, which is robust across all "models only" equations and across the three rating classes, reveals the value added of the MKMV proprietary information. Furthermore, the explanatory power of the MKMV PDs tends to be robust to the inclusion of macroeconomic controls. This is seen in Tables 10-12, which report the results of regressions that

[62] The similarities between the results of the present exercise and the findings of Borio and Lowe (2002) notwithstanding, one should keep in mind that the latter paper builds a non-linear indicator of banking system distress. The length of the sample size employed herein limits the analysis to the linear case.

mimic the specification adopted for the "academic" models.[63] Nevertheless, considering macro variables from the financial and the real sides of the economy substantially improves the goodness of fit measures, especially in the BBB and B rating classes. The MKMV-related results are, however, preliminary because they are based on less than seven years of data that do not encompass a full credit cycle.

Conclusion

The paper uses firm-level data and evaluates six structural credit risk models by examining the overall level and the time path of the PDs they deliver. In contrast to the previous literature, the analysis finds that the PDs implied by some of the models match well the time average of ex post default rates. The models are also in a position to capture, albeit partially, the intertemporal evolution of default rates. The purely theoretical forecasts of credit risk are substantially improved upon by the introduction of macroeconomic variables, which reflect the business and credit cycles.

The best performer among the "academic" frameworks is the "endogenous default" model developed in Leland and Toft (1996). The predictions of that model are quite sensitive to the dynamics of firms' leverage but this sensitivity is borne out by the data. As a result, the LT PDs not only match closely the overall level of default rates but also consistently help to explain their time path.

The ad hoc nature of defaults in the "exogenous default" models is a drag on their performance. The PDs delivered by these models tend to overreact to the level of the default recovery rate, which leads to systematic under-prediction of default rates. Nevertheless, "exogenous default" models do exhibit significant explanatory power in forecasting credit risk.

Future research could provide a useful contribution to the subject matter of this paper by resolving data limitations identified in the main text. These limitation are to be kept in mind when interpreting the results presented above. On the one hand, longer data series, incorporating several credit cycles, would shed further light on the robustness of the models' success in accounting for upturns and downturns in credit risk. On the other hand, larger cross-sections would increase significantly one's confidence in the theoretical forecasts of default rates at different points in time.

Resolving the data limitations would provide solid grounding for research that goes beyond the topic of this paper. The derivation of firm-specific theoretical PDs constitutes a useful first step towards analyses of *portfolio* risk. Such PDs and estimates of default correlations across firms provide sufficient inputs for copula methods, which deliver the probability of any number of defaults in a group of borrowers.

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[63] In particular, the lags of the macroeconomic controls in Tables 10, 11 and 12 are the same as in Tables 7, 8 and 9, respectively.

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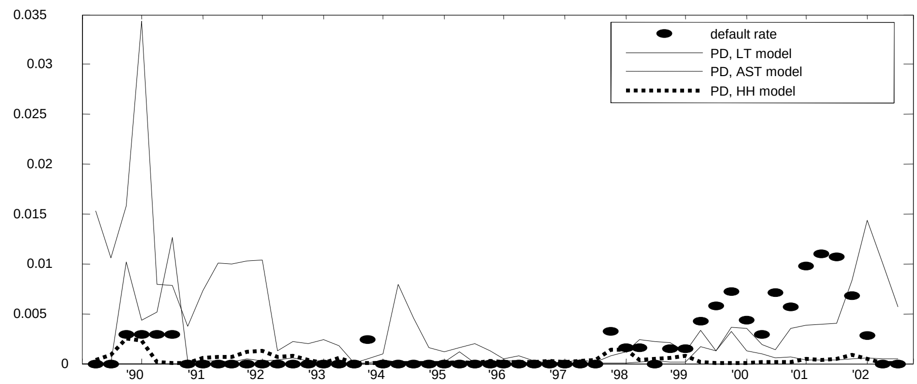
Figures and Tables

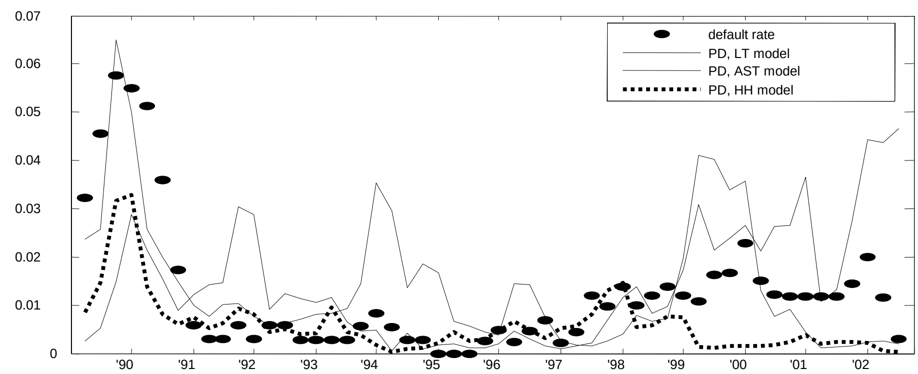
Figure 1

One-year default rates and average model-implied PDs (time varying risk-free rate)

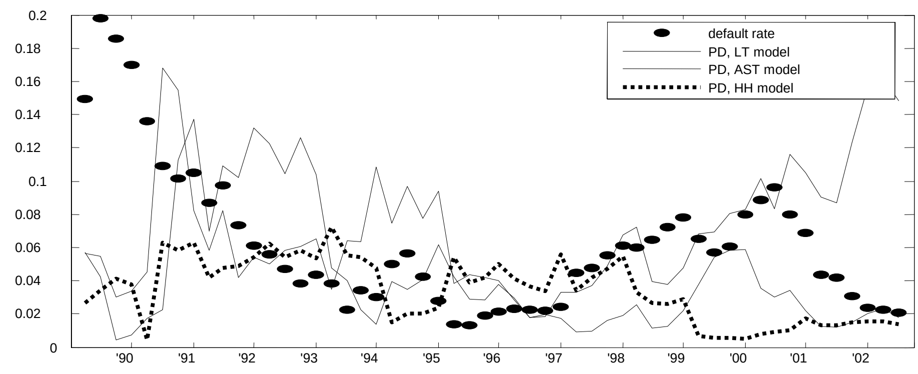
BBB-rated firms



BB-rated firms



B-rated firms

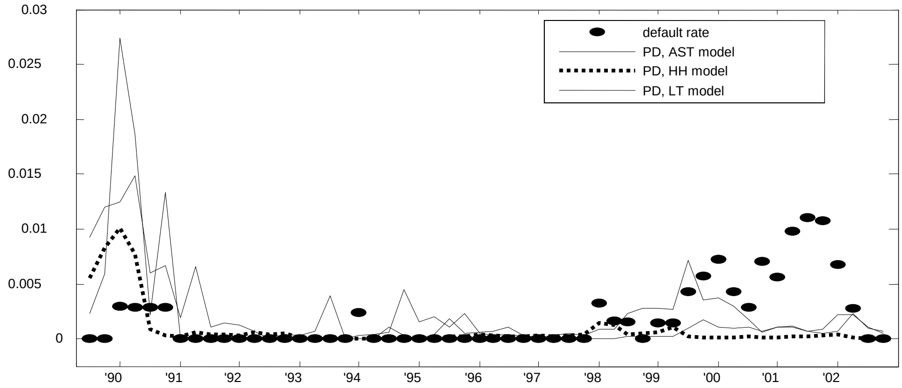


Note: In each quarter, a plotted PD stands for the average of firm-specific model-based PD's. AST refers to Anderson, Sundaresan and Tychon (1996), LT refers to Leland and Toft (1996) and HH refers to Huang and Huang (2003).

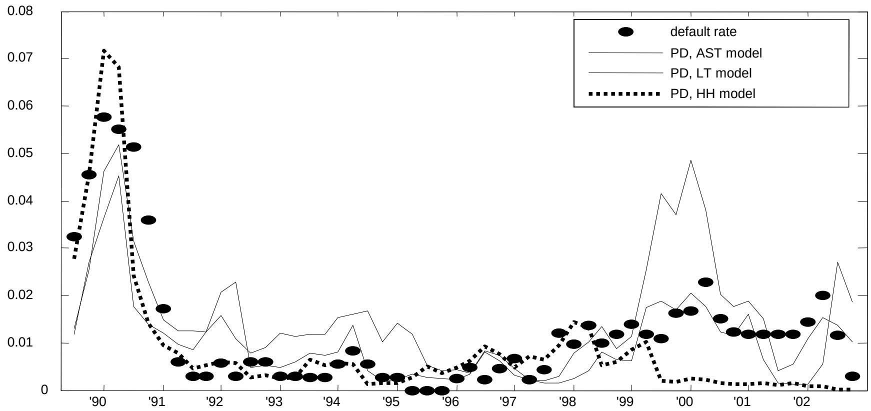
Figure 1a

One-year default rates and average model-implied PDs (constant risk-free rate)

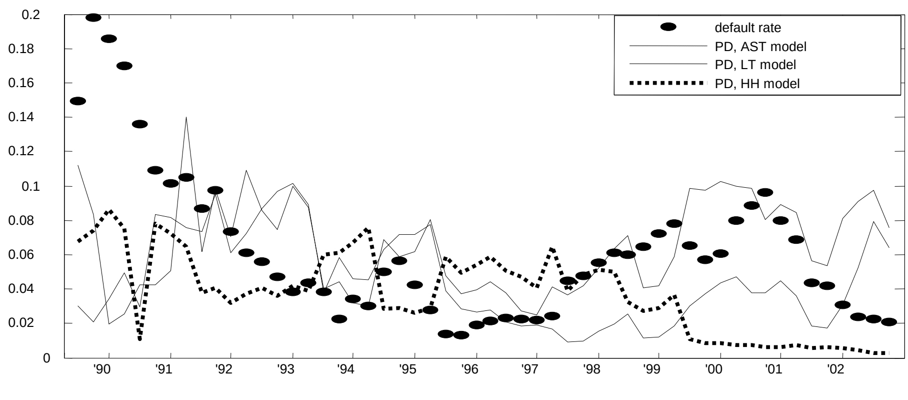
BBB-rated firms



BB-rated firms



B-rated firms



Note: In each quarter, a plotted PD stands for the average of firm-specific model-based PD's. AST refers to Anderson, Sundaresan and Tychon (1996), LT refers to Leland and Toft (1996) and HH refers to Huang and Huang (2003).

Figure 2

Implications of the LT model for firms rated BB in Q4 2001

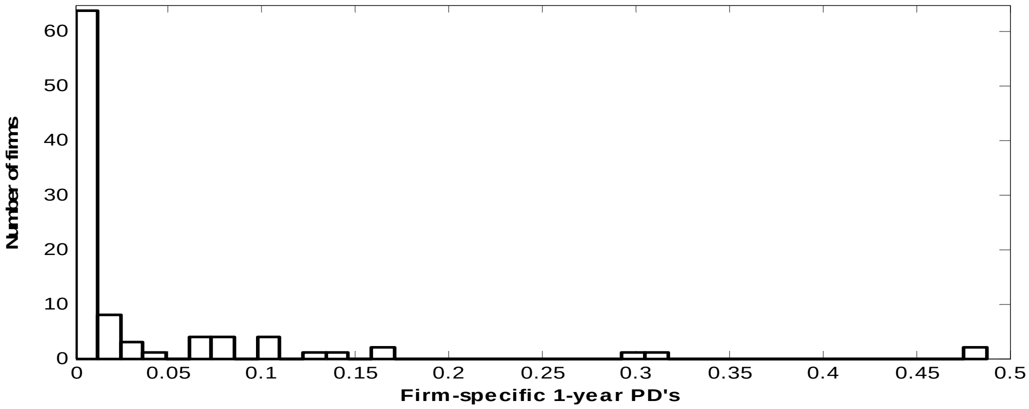
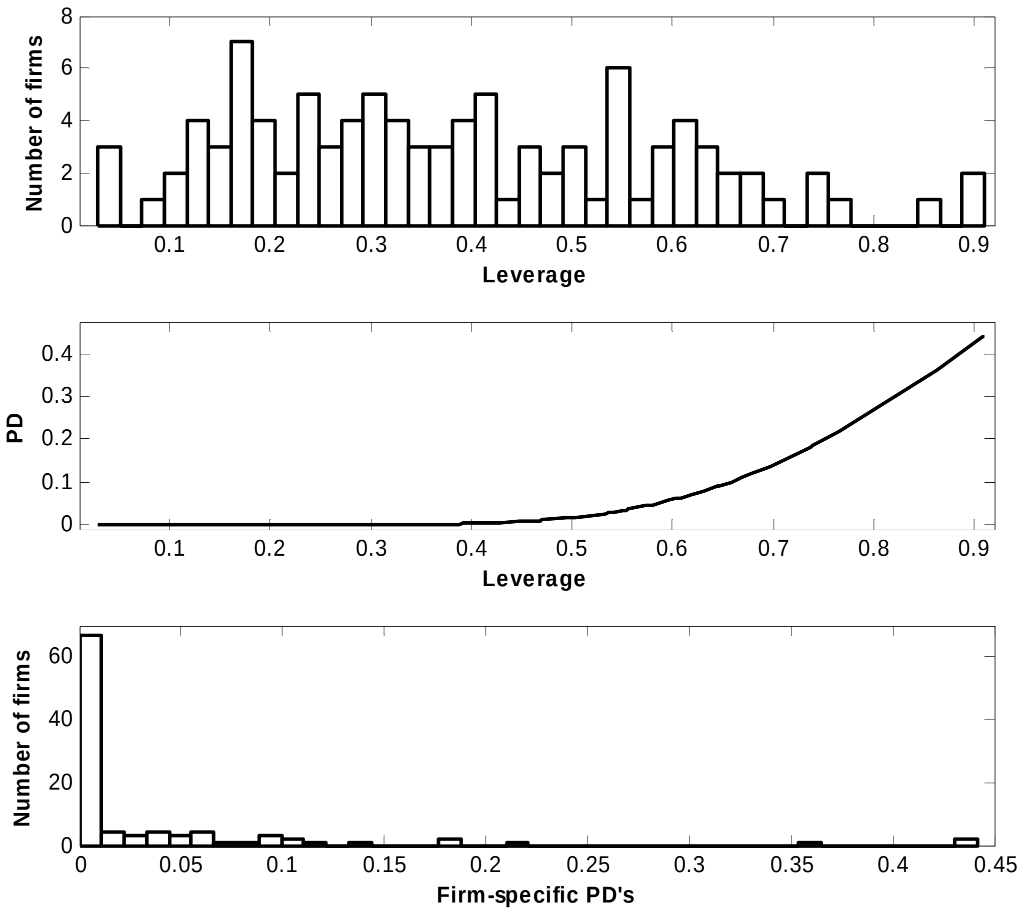


Figure 3

Illustration of the "Jensen inequality" effect (LT model, BB-rated firms, Q4 2001)

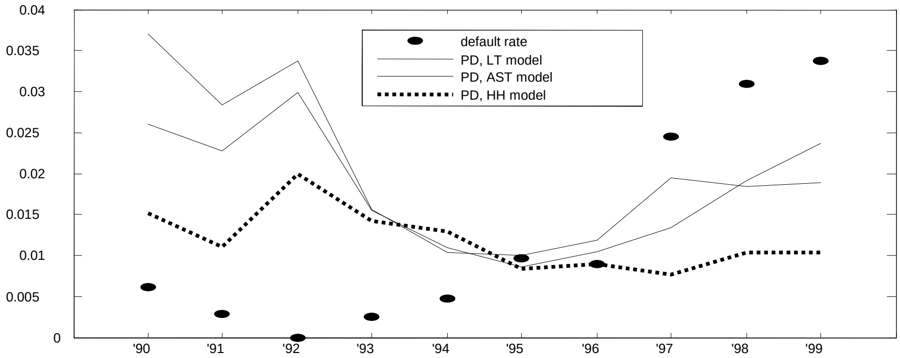


Note: LT refers to Leland and Toft (1996). In Figure 3, the derivation of firm-specific PDs (middle and bottom panel) uses leverage data "as is" but keeps all other parameters at their means in the cross section.

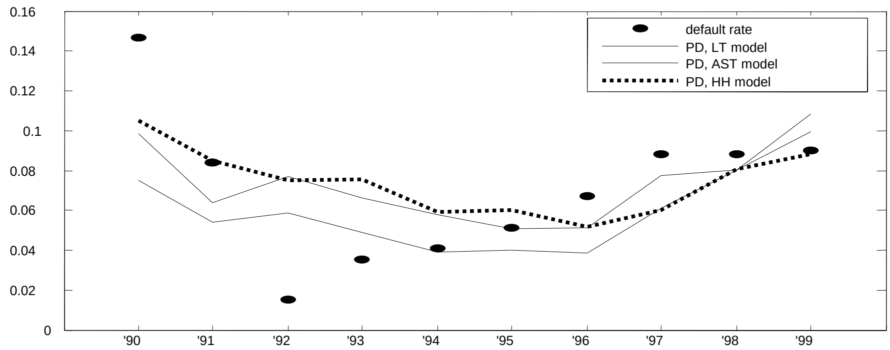
Figure 4

Five-year default rates and average model-implied PDs

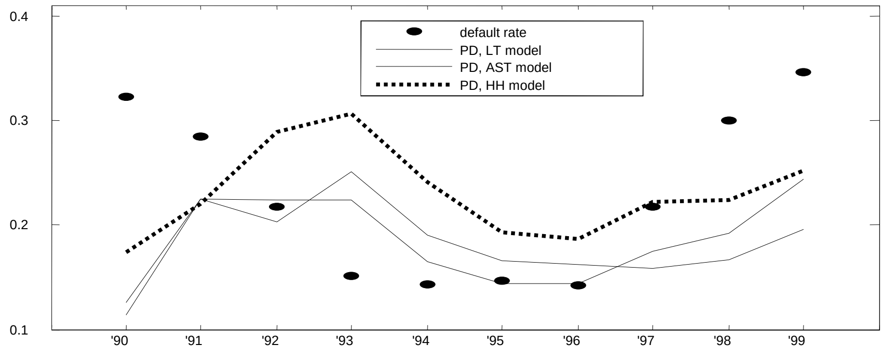
BBB-rated firms



BB-rated firms



B-rated firms



Note: In each quarter, a plotted PD stands for the average of firm-specific model-based PD's. AST refers to Anderson, Sundaresan and Tychon (1996), LT refers to Leland and Toft (1996) and HH refers to Huang and Huang (2003).

Table 1

Model parameters and their calibration[1]

Parameter	Description	Firm specific Y or N	Time varying Y (y or q) or N
c	Coupon rate	Y	Y(y)
T	Time to maturity	Y	Y(y)
r	Risk-free rate of return	N	Y(q) or N
l	Leverage ratio	Y	Y(q)
δ	Asset payout rate	Y	Y(q)
λ	Asset risk premium	Y	Y(q)
σ	Asset volatility	Y	Y(q)
ρ	Default recovery rate	N	Y(y)
α or K	Default cost[2]	N	Y(q) in “endogenous default” models; N in “exogenous default” models
V^*	Default boundary[2]	Y	Y(q)
τ	Tax rate	N	N
m	Monitoring cost	N	N
$k_r, \sigma_r, \bar{r}, \sigma_{rV}$, $k_\lambda, \sigma_\lambda, \bar{\lambda}, \sigma_{\lambda V}$, k_ℓ, ν	Parameters of the second stochastic process [“exogenous default” models only]	N	N

Note: Y = yes, N = no, y = yearly, q = quarterly

[1] Unless stated explicitly otherwise, the parameter values are the same across models and are based on assumptions adopted by Leland (2002) and/or Huang and Huang (2003). [2] The calibrated values of the default boundary and the (dead-weight) default cost depend on the type of the underlying model and on the values of other model parameters. In the “exogenous default” models, α is fixed at 40% and, given an estimate of leverage, V^* is set to be consistent with an estimate of the default recovery rate. In the “endogenous default” models, the values of α (or K) and V^* are determined simultaneously by estimates of the following debt characteristics: coupon rate; time to maturity (LT model only); risk-free rate of return; leverage; asset pay-out rate and volatility; default recovery rate; relevant tax rate (LT model only); and monitoring cost (AST model only).

Table 2

The impact of parameter changes on model-implied PDs[1]

Parameter	Description	Channel of the impact	"exogenous default" models			"endog. def." models	
			CDG	LS	HH	LT	AST
c	Coupon rate	assets process					
		default trigger[2]				+	+
T	Time to maturity	assets process					
		default trigger				–	
r	Risk-free rate of return	assets process	–	–	–	–	–
		default trigger				–	+
ℓ	Leverage ratio[2]	assets process					
		default trigger	+	+	+	+	+
δ	Asset payout rate	assets process	+	+	+	+	+
		default trigger				+	–
λ	Asset risk premium	assets process	–	–	–	–	–
		default trigger					
σ	Asset volatility	assets process	+	+	+	+	+
		default trigger				–	–
ρ	Default recovery rate[2]	assets process					
		default trigger	+	+	+	+	+
α or K	Default cost[2]	assets process					
		default trigger	+	+	+	+	+
τ	Tax rate	assets process					
		default trigger				–	
m	Monitoring cost	assets process					
		default trigger					–
k_ℓ	mean reversion in leverage	assets process	–				
		default trigger					
σ_{rv}	covariance of r and assets[3]	assets process		–			
		default trigger					
$\sigma_{\lambda v}$	covariance of λ and assets[3]	assets process			–		
		default trigger					

[1] Unless explicitly stated otherwise, the table reports the sign of the partial derivative of the PD with respect to the parameter in the row heading. An empty cell signifies "not applicable in the particular model". The last three rows contain key parameters of the second stochastic process of the "exogenous default" models. [2] In all the models, the default trigger and the default cost are calibrated to be consistent with the default recovery rate and leverage. In the "endogenous default" models the default trigger is also influenced by coupon rate; time to maturity; risk-free rate of return; asset pay-out rate and volatility; default recovery rate; relevant tax rate; and monitoring cost. [3] An increase in this parameter increases/decreases the probability that a positive shock to assets raises the PD.

Table 3

Key parameters of the structural credit risk models

	BBB-rated firms[1] (averages)	BB-rated firms[1] (averages)	B-rated firms[1] (averages)	Representative firm[2]
Expected return on assets	10.5%	10.4%	10.2%	12%
Asset payout rate	4%	4.2%	5.8%	6%
Asset volatility	26%	31.2%	32%	23%
Leverage	32%	39%	53.4%	43.3% (BBB) 53.5% (BB) 66% (B)
Coupon rate	7.7%	8.6%	9.8%	8%
Time to maturity	10 years	8.6 years	8.3 years	10 years

[1] Based on the data and calibration methodology described in Sections 2 and 3, respectively. [2] Parameter values adopted by Leland (2002) and/or Huang and Huang (2003).

Table 4

The "Jensen inequality" effect

(1-year default rates vs. 1-year PDs, all numbers in percentage points)

		Average ex post default rates[1,2]	PDs of representative firms[3]	PDs of "average" firms[1,4]	Averages of firm-specific PDs[1,4]
Rating	B	6.2	0.2	0.9	6.5
	BB	1.2	$3.5 * 10^{-3}$	$5 * 10^{-2}$	1.4
	BBB	0.2	$2.7 * 10^{-5}$	$2.3 * 10^{-4}$	0.2

[1] Sample period: Q1 1990 to Q2 2003. [2] Source: Moody's Investors Service. [3] As implied by the Leland (2002) calibration of the Leland and Toft (1996) model. [4] As implied by the model of Leland and Toft (1996) when it is calibrated according to the methodology of Section 3.

Table 5

The "Jensen inequality" effect

(5-year default rates vs. 5-year PDs, all numbers in percentage points)

		Average ex post default rates[1,2]	PDs of representative firms[3]	PDs of "average" firms[1,4]	Averages of firm-specific PDs[1,4]
Rating	B	22.0	12.0	14.6	18.0
	BB	6.9	4.0	4.1	6.0
	BBB	1.2	1.2	0.4	1.8

[1] Sample period: Q1 1990 to Q2 2003. [2] Source: Moody's Investors Service. [3] As implied by the Leland (2002) calibration of the Leland and Toft (1996) model. [4] As implied by the model of Leland and Toft (1996) when it is calibrated according to the methodology of Section 3.

Table 6

Credit Risk Models and Basel II

(all numbers in percentage points)[1]

	BBB					BB					B				
	Descriptive statistics of "optimal" capital														
Mean[2]	**2.7**					**6.7**					**12.5**				
Standard deviation[2]	**2.1**					**2.7**					**2.7**				
	Descriptive statistics of discrepancies between "optimal" capital and model-implied capital														
	LT[3]	**AST[3]**	**HH[3]**	**M KMV[3]**	**SA[4]**	**LT**	**AST**	**HH**	**M KMV**	**SA**	**LT**	**AST**	**HH**	**M KMV**	**SA**
Mean Error	-0.0 *(-0.9)*[2]	0.0 *(-1.9)*	1.0 *(-2.4)*	*(2.2)*	5.3 *(4.3)*	0.6 *(0.3)*	0.4 *(-1.0)*	-1.2 *(-2.6)*	*(2.0)*	1.3 *(0.7)*	0.5 *(1.2)*	-1.2 *(-2.0)*	-2.2 *(-3.2)*	*(0.8)*	-0.5 *(0.1)*
Mean Absolute Error	1.4 *(1.7)*	2.1 *(2.1)*	1.7 *(2.4)*	*(2.5)*	5.3 *(4.3)*	1.7 *(1.5)*	2.4 *(2.1)*	2.4 *(3.7)*	*(2.0)*	2.3 *(1.1)*	2.3 *(1.8)*	2.9 *(2.8)*	3.6 *(4.2)*	*(1.2)*	2.2 *(1.7)*

[1] Main results correspond to the period Q1 1990 to Q2 2003. Numbers in parentheses pertain to the period Q4 1996 to Q2 2003. [2] Based on the IRB regulatory capital implied by actual default rates. [3] Based on the discrepancy between the IRB regulatory capital implied by actual default rates and the IRB regulatory capital implied by the particular model. [4] Based on the discrepancy between the IRB regulatory capital implied by actual default rates and the regulatory capital implied by the standardised approach.

Table 7

BBB-rated firms[1]

Dependent variable: ex post default rate

	"Models only" regressions				**Regressions with model-based PDs and macro variables**						
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Constant	-0.002 (0.11)[2]	-0.001 (0.44)	-0.001 (0.34)	-0.003 (0.13)	**-0.003** (0.02)	**-0.002** (0.12)	**-0.004** (0.02)	-0.001 (0.33)	**0.005** (0.00)	0.000 (0.78)	0.001 (0.62)
ex post default rate (4-quarter lag)	**0.66** (0.02)	**0.63** (0.03)	**0.62** (0.03)	**0.76** (0.01)					**0.49** (0.02)		
PD from LT model[3]	**0.62** (0.08)			**1.58** (0.00)	**0.78** (0.04)	**0.96** (0.02)	**0.73** (0.04)	0.28 (0.28)	0.36 (0.30)	0.21 (0.44)	0.21 (0.42)
PD from AST model[3]		-0.39 (0.47)		***-1.79*** (0.06)	0.84 (0.21)	***-2.24*** (0.01)	-0.20 (0.80)	-0.78 (0.10)	-0.57 (0.36)	-0.66 (0.17)	-0.63 (0.29)
PD from HH model[3]			-0.14 (0.87)	-0.79 (0.58)	***-2.99*** (0.00)	1.33 (0.29)	-1.23 (0.31)	0.45 (0.54)	-0.39 (0.67)	0.35 (0.62)	0.32 (0.73)
Credit/GDP gap (12-quarter lag)					**0.27** (0.00)		**0.21** (0.00)				0.04 (0.36)
Asset price gap (21-quarter lag)						**0.10** (0.00)	**0.04** (0.03)				0.02 (0.15)
GDP gap (8-quarter lag)								**0.36** (0.00)		**0.32** (0.00)	**0.18** (0.06)
Term spread (6-quarter lag)									**-0.46** (0.00)	-0.08 (0.41)	-0.15 (0.13)
adjusted $R^{2\,(4)}$	0.13	0.07	0.07	0.25	0.67	0.51	0.70	0.78	0.64	0.78	0.80

[1] Regressions based on cross-sectional averages of one-year theoretical PDs and one-year ex post default rates. 54 observations from Q1 1990 to Q2 2003. Estimation adopts the Tobit model: there are 32 censored and 22 uncensored observations of the dependent variable. [2] The p-values (in parentheses) are based on Huber-White robust covariance matrices. Entries in bold indicate coefficients that are statistically significant at the 10% level. Italicised entries mark statistically significant coefficients that are of the "wrong" sign. [3] LT refers to Leland and Toft (1996), AST refers to Anderson, Sundaresan and Tychon (1996), and HH refers to Huang and Huang (2003). [4] Reflects the goodness of fit vis-à-vis the latent dependent variable.

Table 8

BB-rated firms[1]

Dependent variable: ex post default rate

	"Models only" regressions				**Regressions with model-based PDs and macro variables**						
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Constant	**0.004** (0.01)[2]	**0.003** (0.21)	**0.007** (0.00)	**0.003** (0.14)	**0.007** (0.00)	**0.003** (0.00)	**0.006** (0.00)	**0.003** (0.01)	**0.007** (0.02)	**0.005** (0.03)	**0.006** (0.00)
ex post default rate (4-quarter lag)		**0.40** (0.08)									
PD from LT model[3]	**0.42** (0.01)			**0.44** (0.00)	**0.17** (0.02)	**0.20** (0.00)	**0.11** (0.06)	**0.18** (0.01)	**0.25** (0.01)	**0.15** (0.02)	**0.11** (0.04)
PD from AST model[3]		0.26 (0.22)		-0.18 (0.17)	***-0.21*** (0.01)	***-0.17*** (0.03)	***-0.20*** (0.02)	0.09 (0.22)	0.01 (0.86)	0.12 (0.12)	***-0.18*** (0.03)
PD from HH model[3]			**0.62** (0.00)	**0.54** (0.00)	**0.62** (0.00)	**0.73** (0.00)	**0.71** (0.00)	**0.59** (0.00)	**0.59** (0.00)	**0.60** (0.00)	**0.71** (0.00)
Credit/GDP gap (18-quarter lag)					**0.19** (0.00)		**0.13** (0.00)				**0.13** (0.00)
Asset price gap (21-quarter lag)						**0.10** (0.00)	**0.06** (0.00)				**0.06** (0.00)
GDP gap (8-quarter lag)								**0.24** (0.00)		**0.19** (0.00)	0.01 (0.74)
Term spread (8-quarter lag)									**-0.28** (0.01)	-0.12 (0.26)	0.01 (0.86)
adjusted R^2	31%	24%	23%	55%	74%	73%	79%	69%	63%	69%	79%

[1] Regressions based on cross-sectional averages of one-year theoretical PDs and one-year ex post default rates. 54 observations from Q1 1990 to Q2 2003. [2] The p-values (in parentheses) are based on Newey-West robust covariance matrices (3 lags). Entries in bold indicate coefficients that are statistically significant at the 10% level. Italicised entries mark statistically significant coefficients that are of the "wrong" sign. [3] LT refers to Leland and Toft (1996), AST refers to Anderson, Sundaresan and Tychon (1996), and HH refers to Huang and Huang (2003).

Table 9

B-rated firms[1]

Dependent variable: ex post default rate

	"Models only" regressions				**Regressions with model-based PDs and macro variables**						
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
Constant	0.02 (0.12)[2]	**0.03** (0.00)	0.02 (0.23)	-0.01 (0.46)	0.02 (0.16)	-0.03 (0.11)	0.004 (0.80)	0.003 (0.85)	0.02 (0.09)	0.01 (0.53)	0.02 (0.15)
ex post default rate (4-quarter lag)	**0.37** (0.06)	**0.45** (0.04)	**0.46** (0.01)	**0.39** (0.02)		**0.29** (0.04)		**0.40** (0.00)	**0.55** (0.00)	**0.45** (0.00)	
PD from LT model[3]	0.13 (0.61)			**0.73** (0.00)	**0.39** (0.03)	**0.59** (0.00)	**0.34** (0.03)	0.01 (0.96)	0.26 (0.11)	0.04 (0.79)	-0.09 (0.57)
PD from AST model[3]		-0.26 (0.21)		***-0.57*** (0.00)	***-0.61*** (0.01)	-0.24 (0.26)	-0.34 (0.18)	**0.33** (0.01)	0.00 (0.98)	**0.28** (0.04)	0.19 (0.33)
PD from HH model[3]			0.13 (0.69)	**0.52** (0.05)	**0.94** (0.01)	**0.80** (0.01)	**1.09** (0.00)	**0.37** (0.07)	0.27 (0.11)	**0.32** (0.07)	**0.77** (0.01)
Credit/GDP gap (21-quarter lag)					**1.04** (0.00)		**0.84** (0.01)				**0.73** (0.00)
Asset price gap (16-quarter lag)						**0.34** (0.00)	**0.27** (0.00)				**0.11** (0.09)
GDP gap (4-quarter lag)								**1.44** (0.00)		**1.05** (0.07)	**1.14** (0.02)
Term spread (2-quarter lag)									**-1.69** (0.00)	-0.57 (0.45)	0.06 (0.91)
adjusted R^2	27%	29%	26%	40%	54%	51%	61%	74%	70%	74%	76%

[1] Regressions based on cross-sectional averages of one-year theoretical PDs and one-year ex post default rates. 54 observations from Q1 1990 to Q2 2003. [2] The p-values (in parentheses) are based on Newey-West robust covariance matrices (3 lags). Entries in bold indicate coefficients that are statistically significant at the 10% level. Italicised entries mark statistically significant coefficients that are of the "wrong" sign. [3] LT refers to Leland and Toft (1996), AST refers to Anderson, Sundaresan and Tychon (1996), and HH refers to Huang and Huang (2003).

Table 10

BBB-rated firms[1]

Dependent variable: ex post default rate

	"Models only" regressions		**Regressions with model-based PDs and macro variables**						
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Constant	**-0.005** (0.05)[2]	**-0.0004** (0.03)	**-0.016** (0.00)	-0.004 (0.21)	**-0.013** (0.01)	0.0002 (0.88)	0.002 (0.40)	0.002 (0.20)	0.003 (0.20)
ex post default rate (4-quarter lag)									
PD from LT model[3]		-1.22 (0.10)							
PD from AST model[3]		1.04 (0.65)							
PD from HH model[3]		3.95 (0.20)							
PD, MKMV model[3]	**1.11** (0.00)	**1.56** (0.00)	**1.29** (0.00)	0.68 (0.11)	**0.96** (0.03)	-0.18 (0.46)	**0.73** (0.00)	-0.05 (0.81)	-0.80 (0.80)
Credit/GDP gap (12-quarter lag)			**0.40** (0.00)		**0.37** (0.00)				-0.02 (0.62)
Asset price gap (21-quarter lag)				0.04 (0.16)	0.02 (0.23)				0.02 (0.10)
GDP gap (8-quarter lag)						**0.39** (0.00)		**0.30** (0.00)	**0.24** (0.00)
Term spread (6-quarter lag)							**-0.45** (0.00)	**-0.18** (0.05)	**-0.23** (0.01)
adjusted R^{2} [4]	0.39	0.42	0.84	0.41	0.83	0.79	0.68	0.81	0.81

[1] Regressions based on cross-sectional averages of one-year theoretical PDs and one-year ex post default rates. 27 observations from Q4 1990 to Q2 2003. Estimation adopts the Tobit model: there are 10 censored and 17 uncensored observations of the dependent variable. [2] The p-values (in parentheses) are based on Huber-White robust covariance matrices. Entries in bold indicate coefficients that are statistically significant at the 10% level. Italicised entries mark statistically significant coefficients that are of the "wrong" sign. [3] LT refers to Leland and Toft (1996), AST refers to Anderson, Sundaresan and Tychon (1996), HH refers to Huang and Huang (2003), and M KMV refers to Moody's KMV. [4] Reflects the goodness of fit vis-à-vis the latent dependent variable.

Table 11

BB-rated firms[1]

	"Models only" regressions		Regressions with model-based PDs and macro variables						
Dependent variable: ex post default rate	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Constant	0.002 (0.27)[2]	0.004 (0.12)	0.005 (0.10)	**0.004** (0.06)	**0.005** (0.09)	**0.004** (0.04)	**0.006** (0.05)	**0.006** (0.00)	**0.006** (0.01)
ex post default rate (4-quarter lag)									
PD from LT model[3]		0.05 (0.72)							
PD from AST model[3]		***-0.29*** (0.03)							
PD from HH model[3]		-0.18 (0.33)							
PD, MKMV model[3]	**0.39** (0.00)	**0.45** (0.00)	**0.25** (0.07)	**0.27** (0.00)	**0.23** (0.08)	**0.27** (0.00)	**0.32** (0.00)	**0.24** (0.00)	**0.25** (0.02)
Credit/GDP gap (18-quarter lag)			0.07 (0.14)		0.07 (0.38)				-0.02 (0.72)
Asset price gap (21-quarter lag)				**0.04** (0.03)	**0.03** (0.05)				0.008 (0.72)
GDP gap (8-quarter lag)						**0.18** (0.00)		**0.15** (0.00)	**0.15** (0.04)
Term spread (8-quarter lag)							**-0.24** (0.09)	-0.17 (0.20)	-0.17 (0.34)
adjusted R^2	0.53	0.56	0.57	0.59	0.59	0.64	0.59	0.67	0.64

[1] Regressions based on cross-sectional averages of one-year theoretical PDs and one-year ex post default rates. 27 observations from Q4 1990 to Q2 2003. [2] The p-values (in parentheses) are based on Newey-West robust covariance matrices (2 lags). Entries in bold indicate coefficients that are statistically significant at the 10% level. Italicised entries mark statistically significant coefficients that are of the "wrong" sign. [3] LT refers to Leland and Toft (1996), AST refers to Anderson, Sundaresan and Tychon (1996), HH refers to Huang and Huang (2003), and M KMV refers to Moody's KMV.

Table 12

B-rated firms[1]

Dependent variable: ex post default rate

	"Models only" regressions		**Regressions with model-based PDs and macro variables**						
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Constant	-0.01 (0.37)[2]	-0.02 (0.11)	-0.03 (0.16)	-0.01 (0.63)	-0.03 (0.16)	**0.02** (0.01)	**0.03** (0.01)	**0.02** (0.01)	**0.04** (0.00)
ex post default rate (4-quarter lag)									
PD from LT model[3]		0.13 (0.50)							
PD from AST model[3]		***-0.75*** (0.00)							
PD from HH model[3]		0.10 (0.45)							
PD, MKMV model[3]	**1.04** (0.00)	**1.29** (0.00)	**1.34** (0.00)	**0.82** (0.01)	**1.14** (0.00)	**0.41** (0.00)	**0.63** (0.00)	**0.42** (0.00)	**0.23** (0.02)
Credit/GDP gap (21-quarter lag)			-0.28 (0.25)		-0.34 (0.12)				0.11 (0.23)
Asset price gap (16-quarter lag)				0.17 (0.17)	0.20 (0.14)				0.06 (0.22)
GDP gap (4-quarter lag)						**1.18** (0.00)		**1.03** (0.00)	**0.88** (0.00)
Term spread (2-quarter lag)							**-1.36** (0.00)	-0.24 (0.43)	-0.50 (0.16)
adjusted R^2	0.51	0.76	0.54	0.54	0.58	0.92	0.83	0.92	0.93

[1] Regressions based on cross-sectional averages of one-year theoretical PDs and ex post default rates. 27 observations from Q4 1990 to Q2 2003. [2] The p-values (in parentheses) are based on Newey-West robust covariance matrices (2 lags). Entries in bold indicate coefficients that are statistically significant at the 10% level. Italicised entries mark statistically significant coefficients that are of the "wrong" sign. [3] LT refers to Leland and Toft (1996), AST refers to Anderson, Sundaresan and Tychon (1996), HH refers to Huang and Huang (2003), and M KMV refers to Moody's KMV.